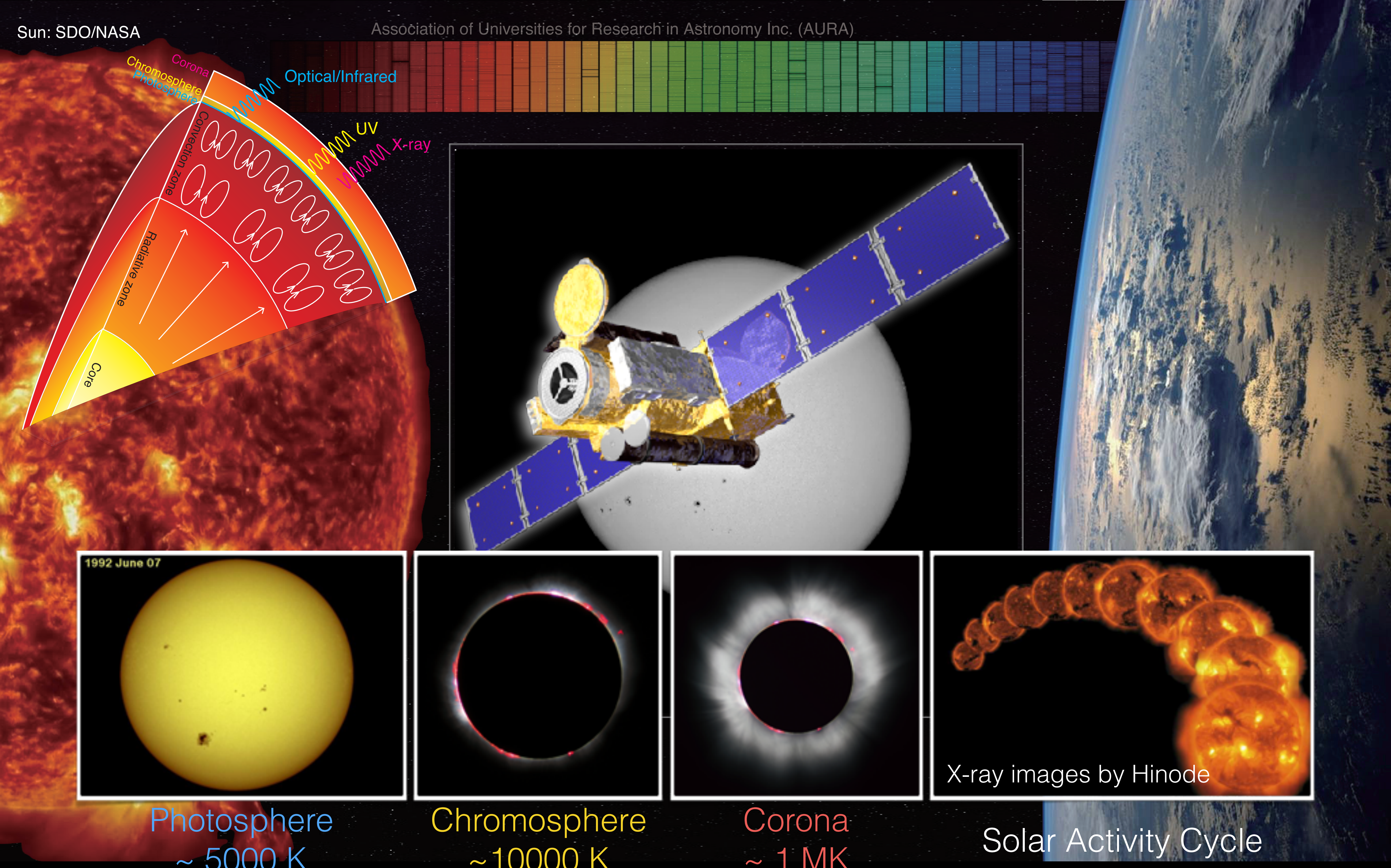
Vortices in the Solar Atmosphere (and Accretion Disks)

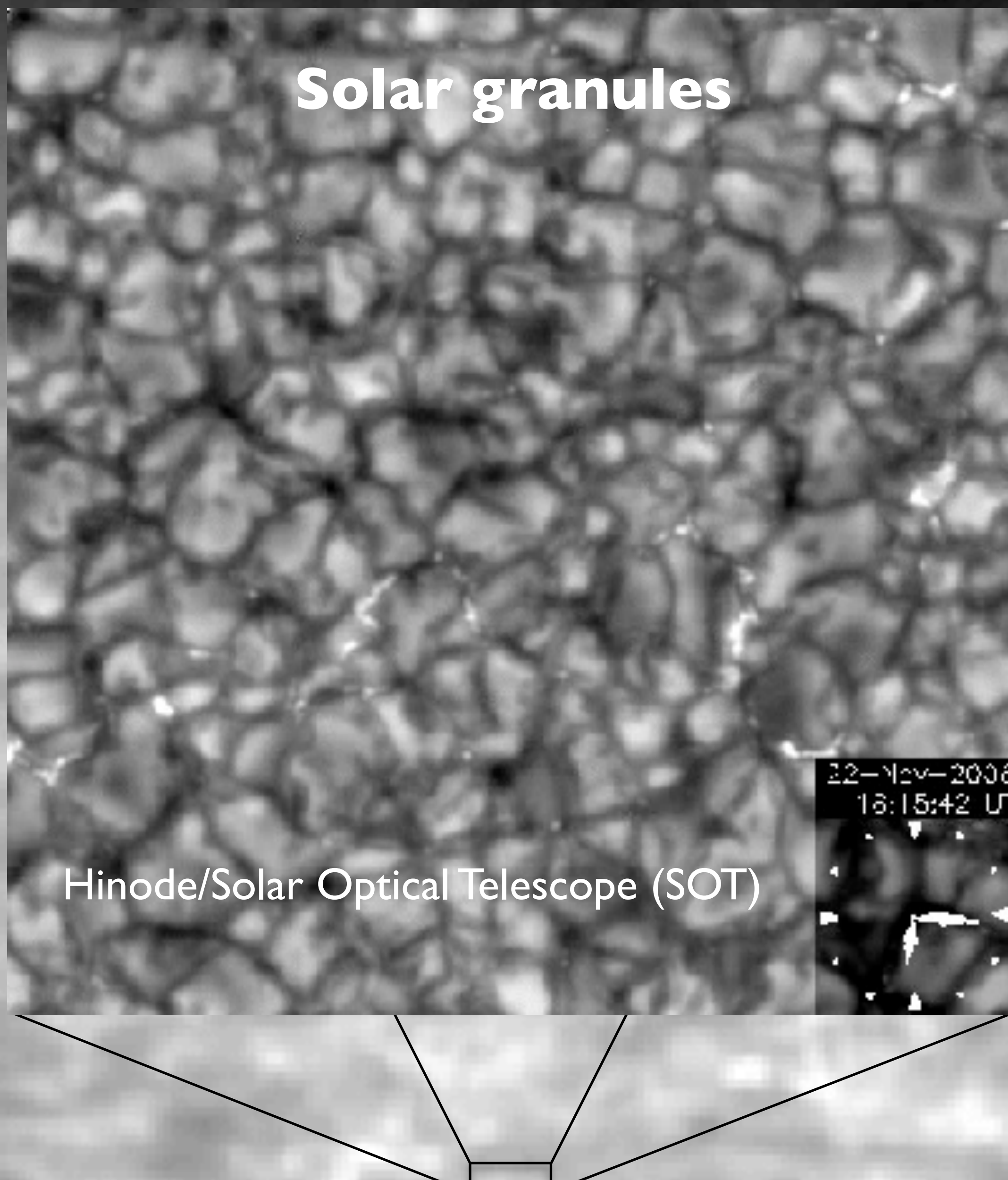
Yoshiaki Kato
(RIKEN)



Structure and Dynamics of the Solar Atmosphere

Solar Chromosphere

Spicules = Dynamical fibrils



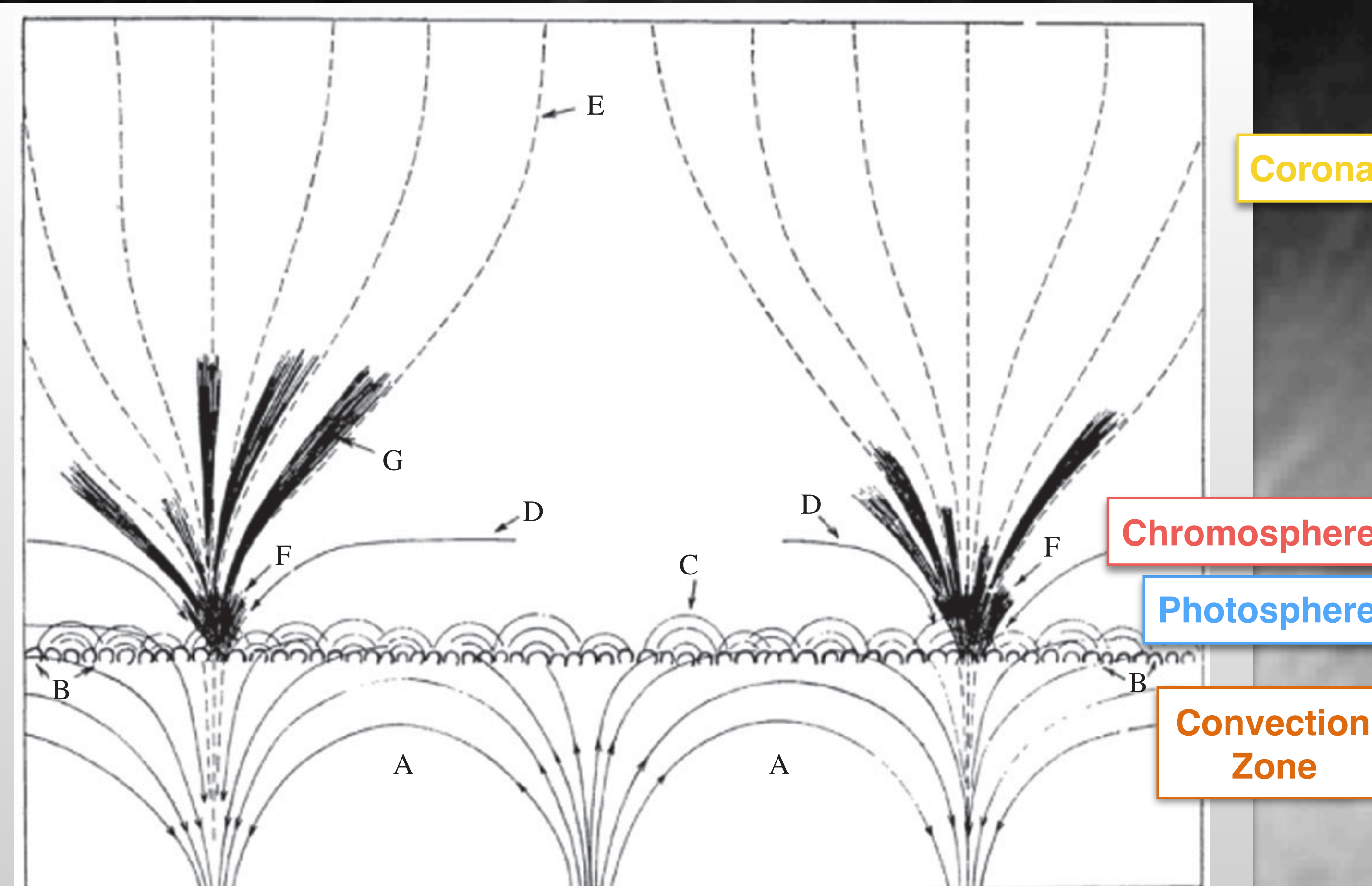


Figure 1. Sketch of the granulation–supergranulation–spicule complex in cross section. A, Flow lines of a supergranulation cell. B, Photospheric granules. C, Wave motions. D, Large-scale chromospheric flow field seen in Hα. E, [Magnetic] lines of force, pictured as uniform in the corona but concentrated at the boundaries of the supergranules in the photosphere and chromosphere. F, Base of a spicule 'bush' or 'rosette', visible as a region of enhanced emission in the Hα and K-line cores. G, Spicules. [...] The distance between the bushes is 30 000 km. Reproduced with permission from Noyes [15], including this caption.

Rutten 2012

Hinode/Ca II H BFI movie
(courtesy by Mats Carlsson)

National Astronomical Observatory of Japan

SOLAR AND ASTROPHYSICS LAB

Radiation MHD equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v})$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{B^2}{2\mu} \right) \boldsymbol{I} - \frac{\boldsymbol{B}\boldsymbol{B}}{\mu} \right] + \rho \boldsymbol{g}$$

$$\frac{\partial e}{\partial t} = -\nabla \cdot \left[\left(\epsilon + p + \frac{B^2}{2\mu} \right) \boldsymbol{v} - \frac{\boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})}{\mu} \right] + \rho \boldsymbol{v} \cdot \boldsymbol{g} + Q$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \qquad \boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} + \eta \boldsymbol{J} \qquad \boldsymbol{J} = \frac{\nabla \times \boldsymbol{B}}{\mu}$$

$$e = \rho \epsilon + \frac{1}{2}\rho v^2 + \frac{B^2}{2\mu}$$

$p = p(\rho, \varepsilon)$ in LTE and Non-LTE

$$Q = Q_{\mathrm{rad}} + Q_{\mathrm{cond}}$$

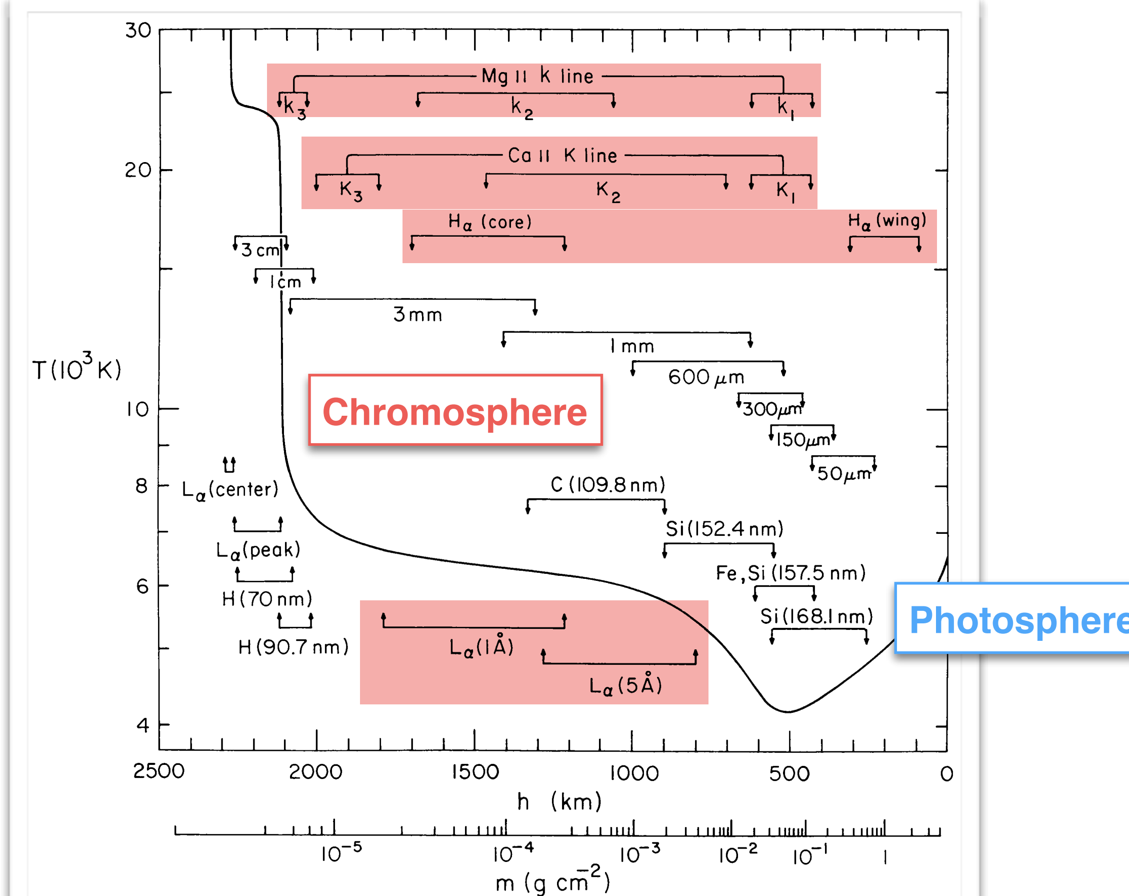
Radiation transfer equation
Population density equation

$$\frac{dI}{d\tau} = S - I$$

$$S_l = \frac{n_j A_{ji} \Psi}{n_i B_{ij} \Phi - n_j B_{ji} \Psi_{se}}$$

$$\frac{Dn_i}{Dt} = \sum_{j \neq i}^{N} n_j P_{ji} - n_i \sum_{j \neq i}^{N} P_{ij}$$

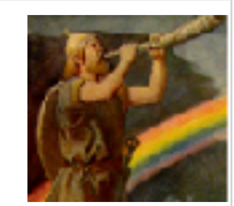
P_{ij} is the probability for a transition from level i to level j



Vernazza, Avrett, Loeser 1981

Empirical radiative loss in the chromosphere: 2.5 - 4.3 kWm^{-2} (Ulmschneider 1974; Vernazza et al. 1981)

Bifrost code
(Gudiksen et al. 2011)



- Finite-difference MHD solver
- Realistic EOS with Hydrogen ionisation
- Radiative transfer:
  - Optically thin radiative transfer (mostly in the corona),
  - Chromospheric radiation approximation (Leenaarts et al. 2007),
  - Full radiative transfer in the photosphere.
- Thermal conduction in the lower corona

Modeling a magnetic flux tube/sheath

for investigating dynamical fibrils

Current sheets will form at the boundary of field lines.

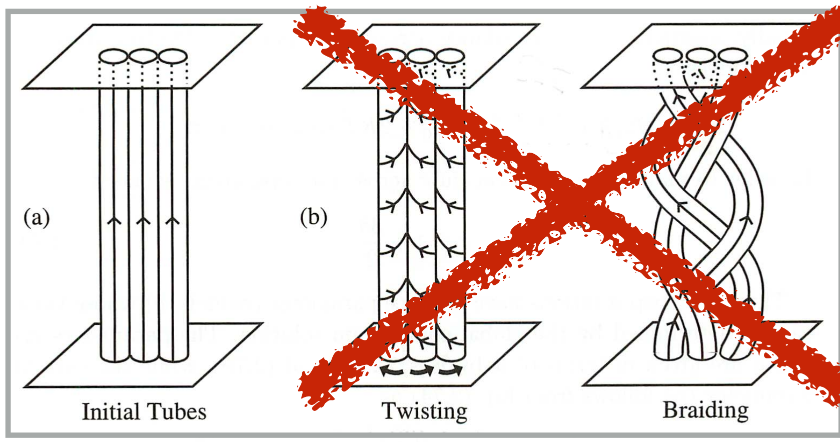
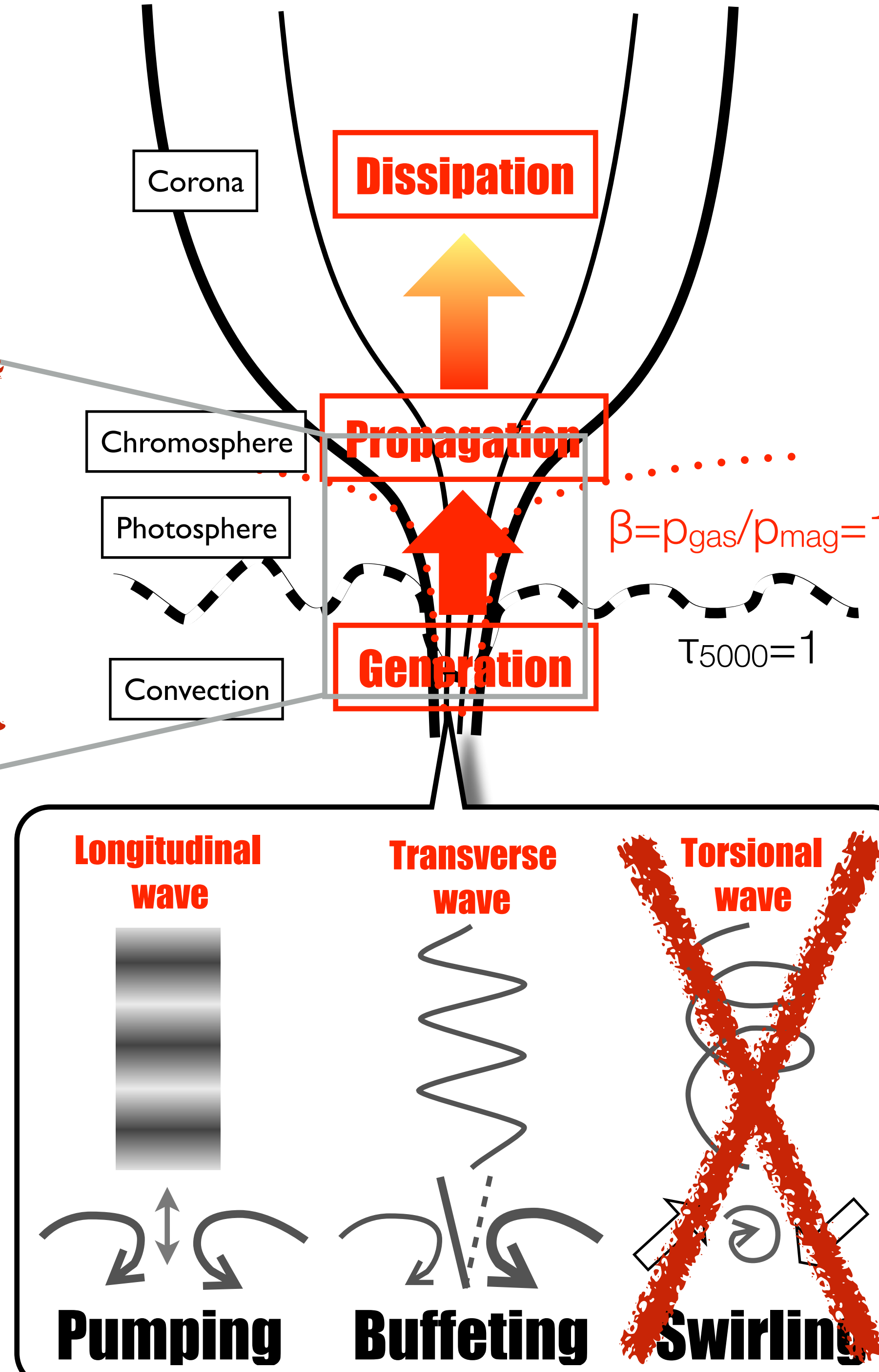


Fig. 2.16. The effect on (a) an initial field of (b) twisting and (c) braiding motions.

Parker 1979, 1990

Magnetic reconnections

Two-dimensional RMHD simulations



Rae & Roberts 1982

$$\frac{\partial^2 Q}{\partial t^2} - c_T^2 \frac{\partial^2 Q}{\partial z^2} + \omega_v^2 Q = 0$$ Klein-Gordon Equation

Plane-parallel wave solution: $Q(z,t) \propto e^{i(\omega t - kz)}$

where $\omega_v^2 = N_0 + \left(\frac{3}{4}-\frac{1}{\gamma}\right)^2 \frac{c_T}{H^2}$, $Q(z,t) = e^{-z/4H} v(z,t)$

the Brunt-Väisälä frequency: $N_0^2(z) = -g\left(\frac{1}{\rho_0}\frac{d\rho_0}{dz} + \frac{g}{c_0^2}\right)$

the tube speed: $c_T = \frac{c_s c_A}{\sqrt{c_s^2 + c_A^2}}$

$$\omega^2 = k^2 c_T^2 + \omega_v^2$$

$$k^2 = \frac{\omega^2 - \omega_v^2}{c_T^2}, \quad \omega = \pm\omega(k) = \pm\left(\omega_v^2 + k^2 c_T^2\right)^{1/2}$$

$\omega < \omega_v$: evanescent $\omega > \omega_v$: wave propagation

Two possible solutions:

1. Forced oscillations above the cutoff frequency
   ➡ Continuous driving
2. Resonant oscillations near the cutoff frequency
   ➡ Impulsive driving

“Realistic” modeling of the solar atmosphere by using Bifrost

Bifrost code
(Gudiksen et al. 2011)

- Finite-difference MHD solver
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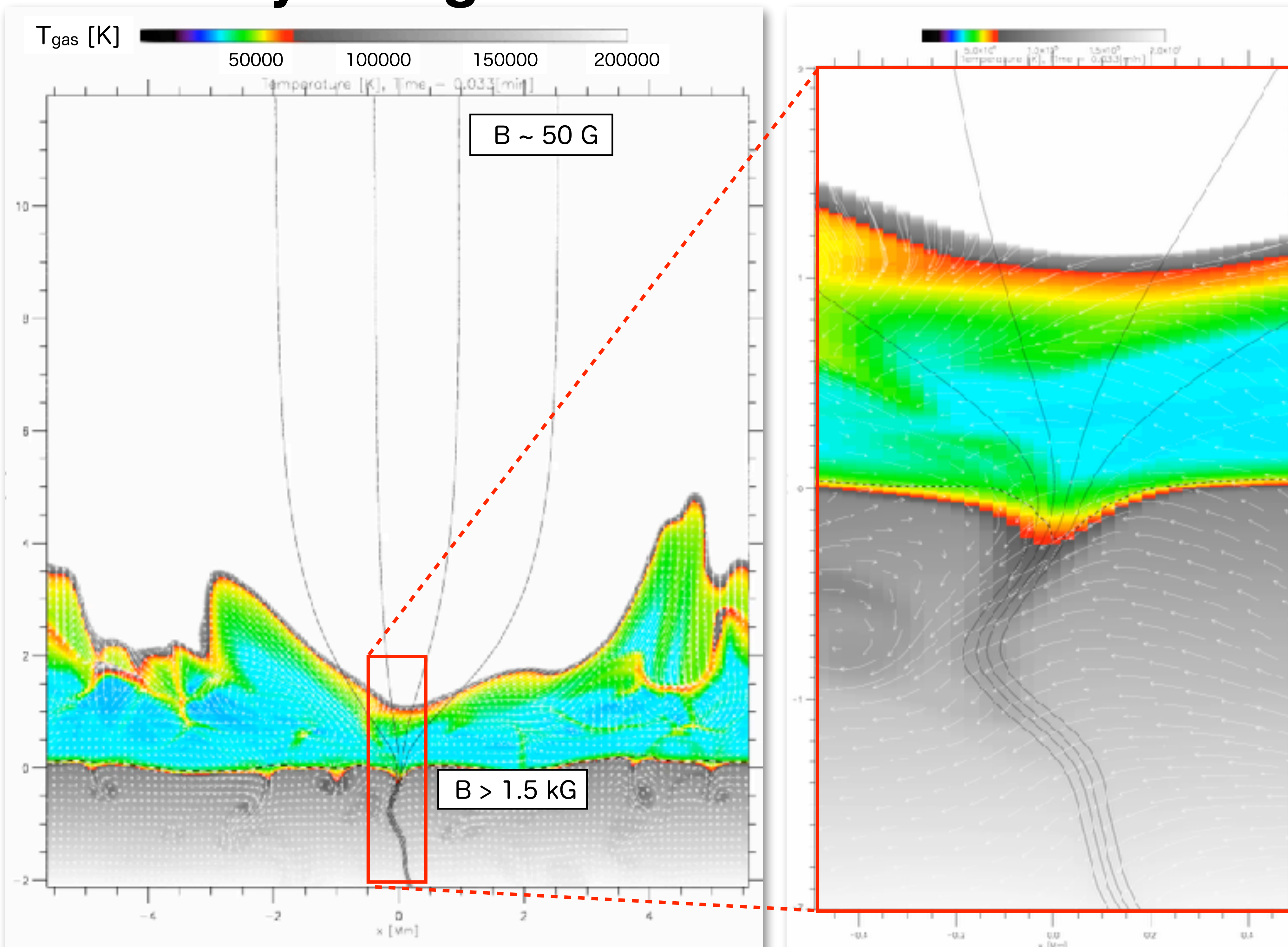
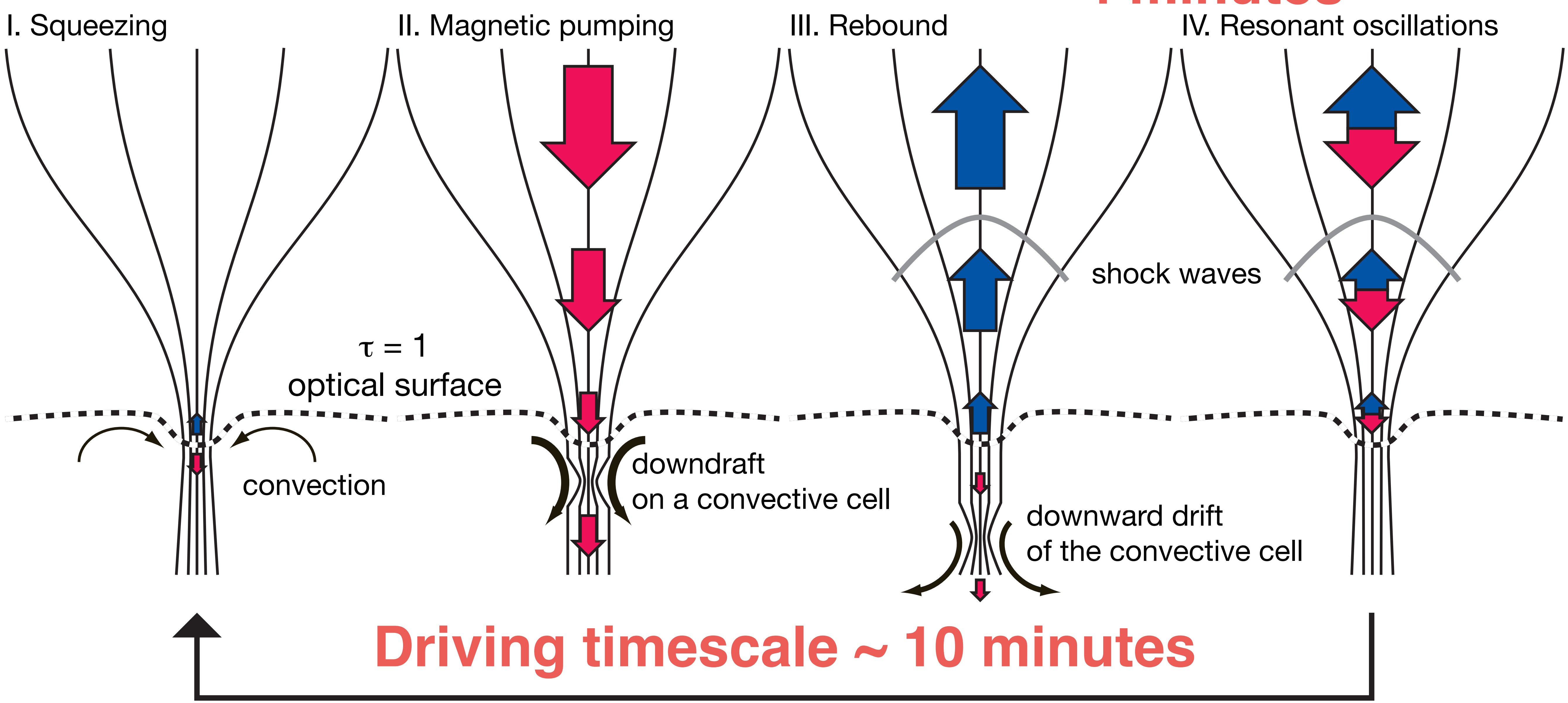


ΔX=28km
N_x=400 N_z=535

Magnetic pumping mechanism

for sustaining the solar chromosphere

Chromospheric response ~ 4 minutes



Summary

Magnetic pumping is responsible for “dynamic fibrils”

Current sheets will form at the boundary of field lines.

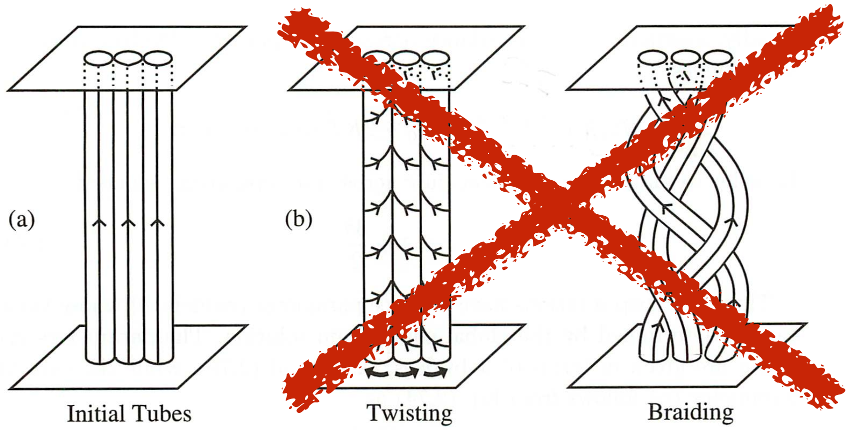
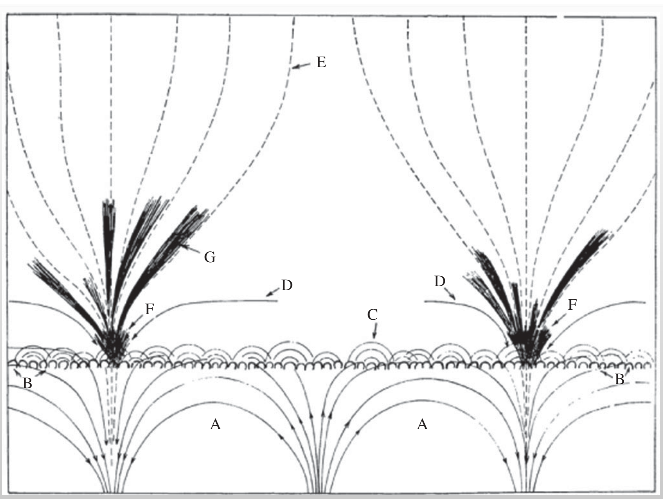
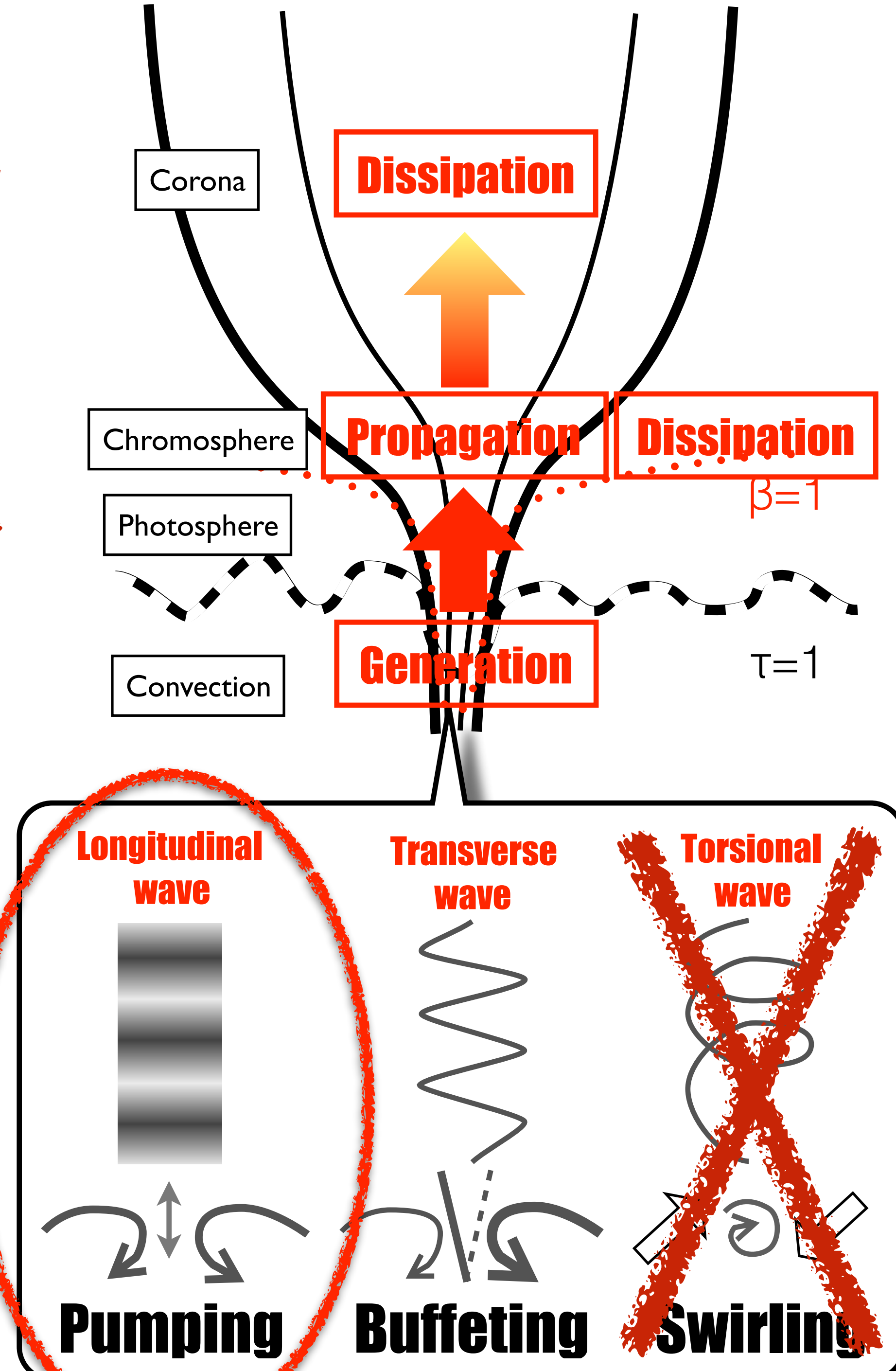


Fig. 2.16. The effect on (a) an initial field of (b) twisting and (c) braiding motions.



Rutten 2012

Recurrent shock waves can sustain solar chromosphere



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References

- Kato, Y., Steiner, O., Steffen, M., Suematsu, Y., *Excitation of slow-modes in network magnetic elements through magnetic pumping*, 2011, ApJ, **730**, L24-L28
- Kato, Y., Steiner, O., Hansteen, V., Gudiksen, B., Wedemeyer, B., Carlsson, M., *Chromospheric and Coronal Wave Generation in a Magnetic Flux Sheath*, 2016, ApJ, **827**, 7-23
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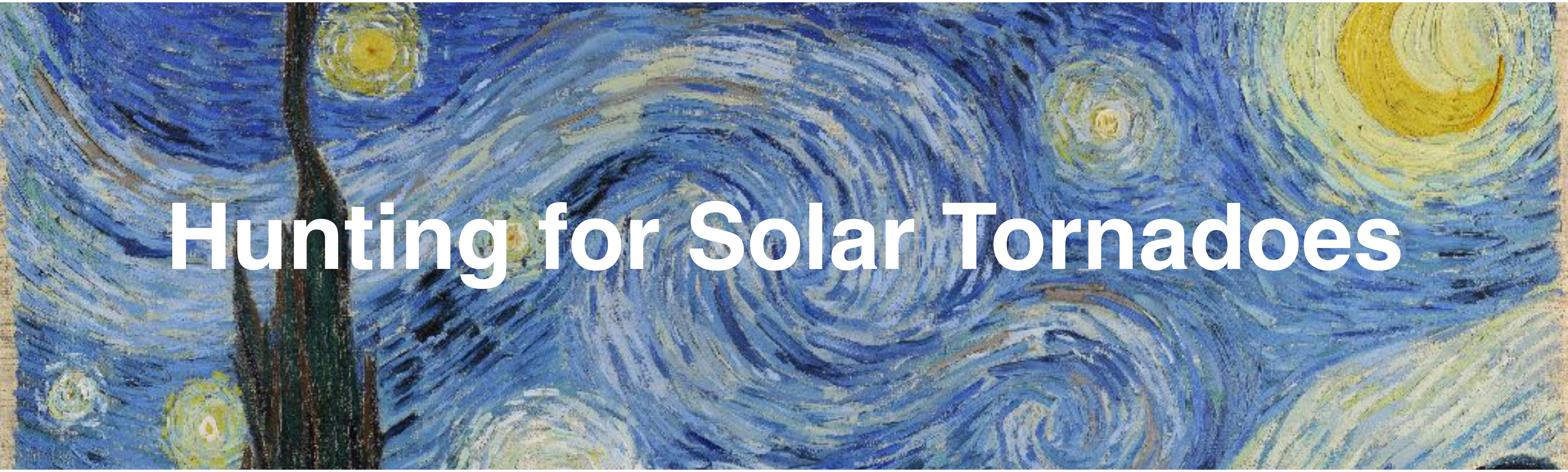


Hunting for Solar Tornadoes

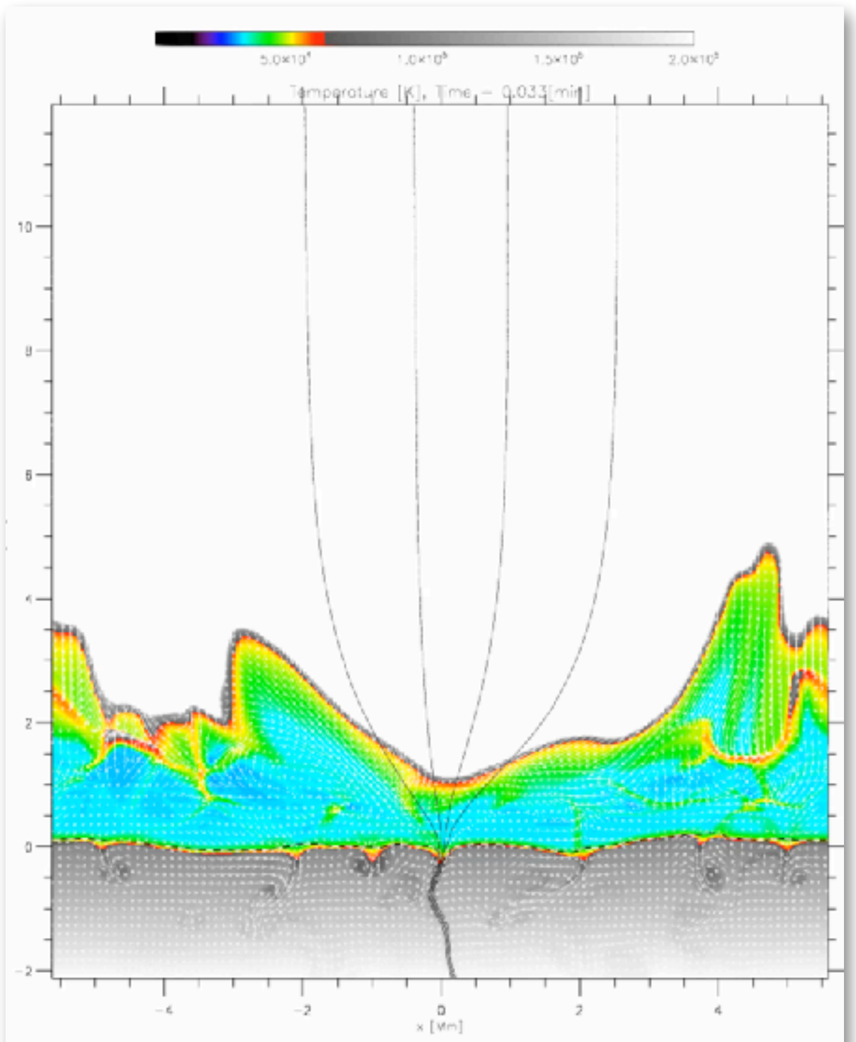
Vincent van Gogh's *The Starry Night (1889)*
the view from the window of his asylum room at Saint-Rémy-de-Provence

Yoshiaki Kato
(University of Oslo —> RIKEN)

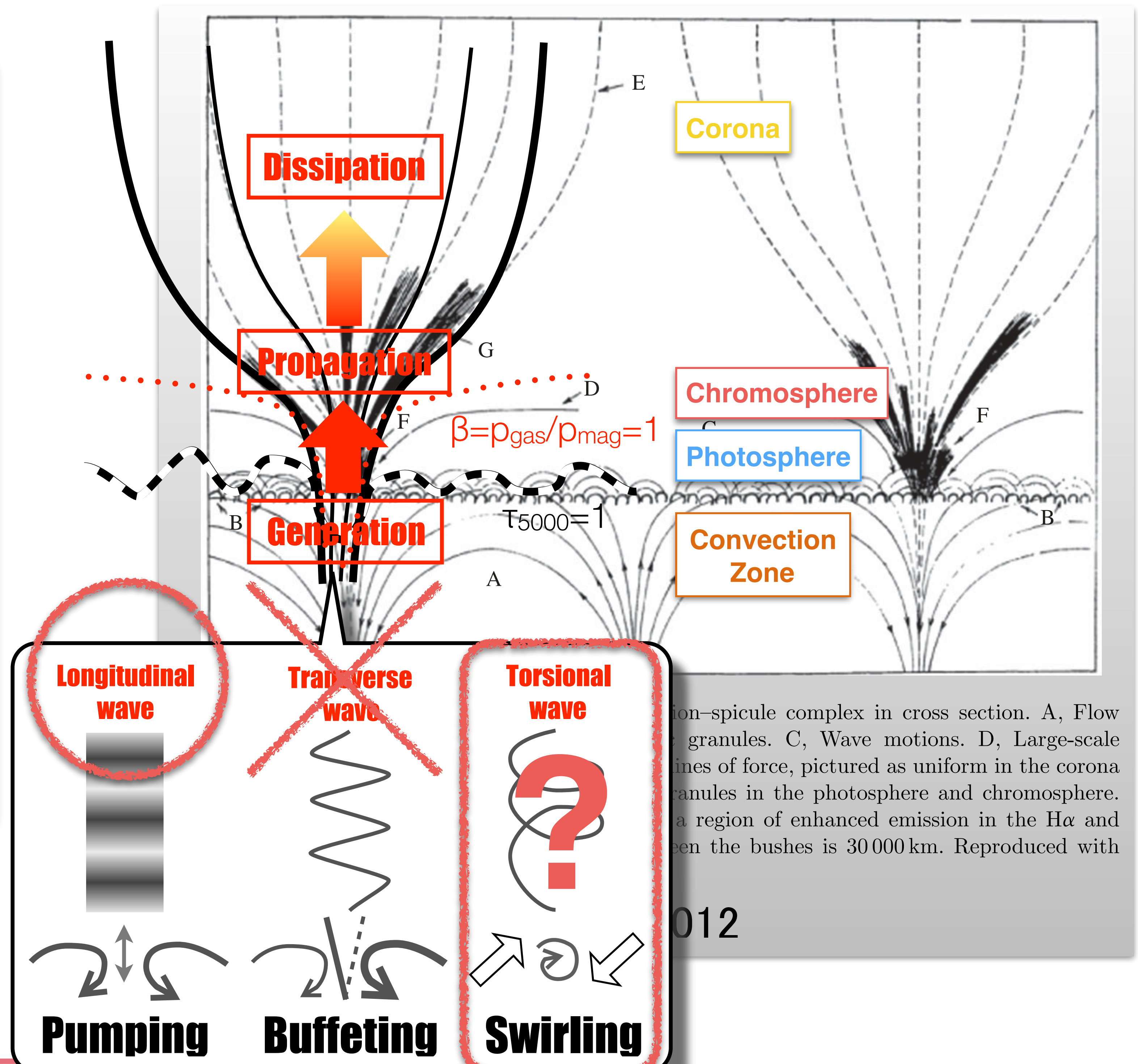
Sven Wedemeyer
(University of Oslo)

Magnetic flux tube

a magnetic portal for energy transport

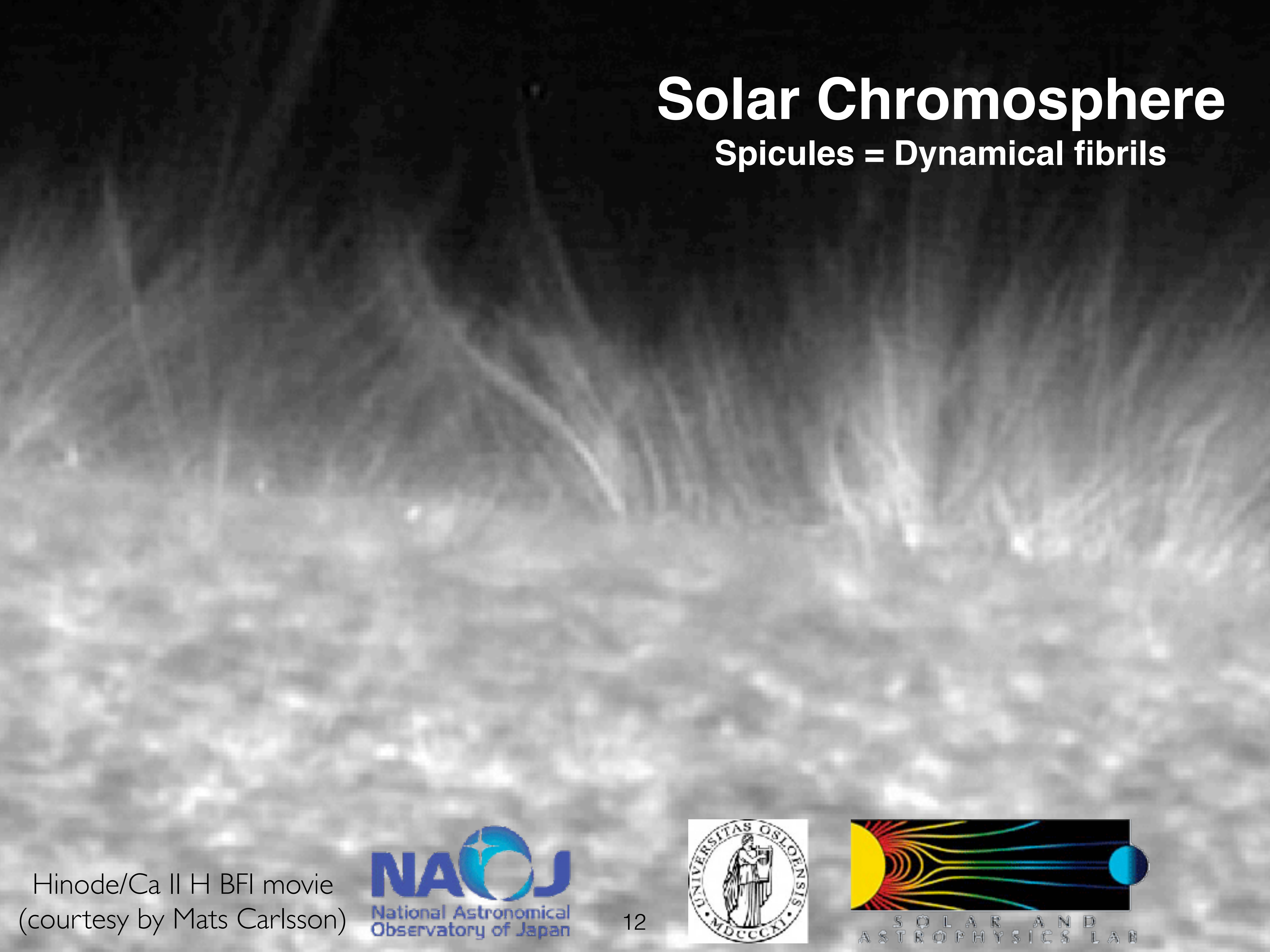


YK et al. 2016



Solar Chromosphere

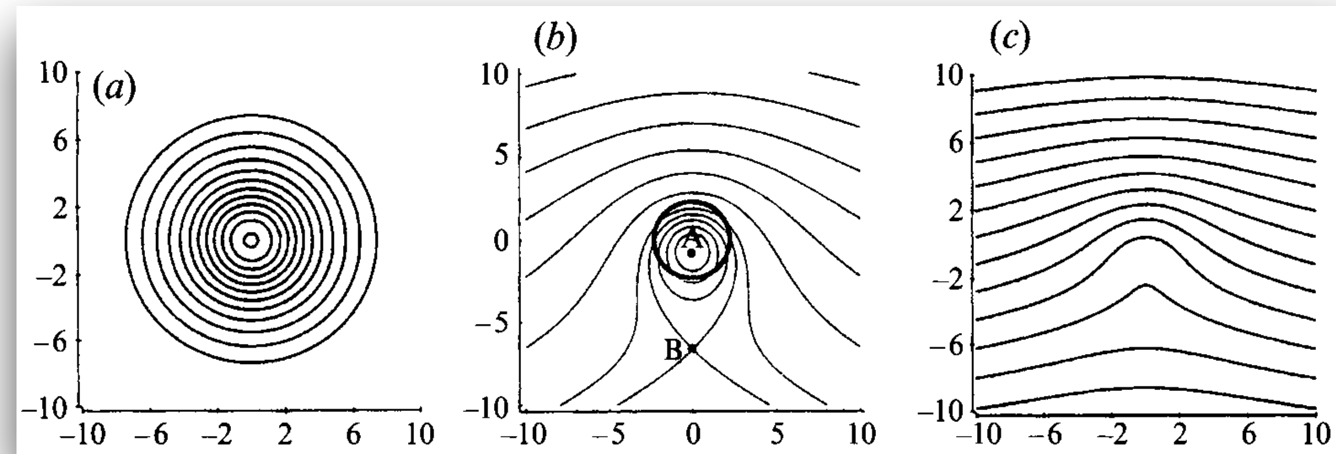
Spicules = Dynamical fibrils



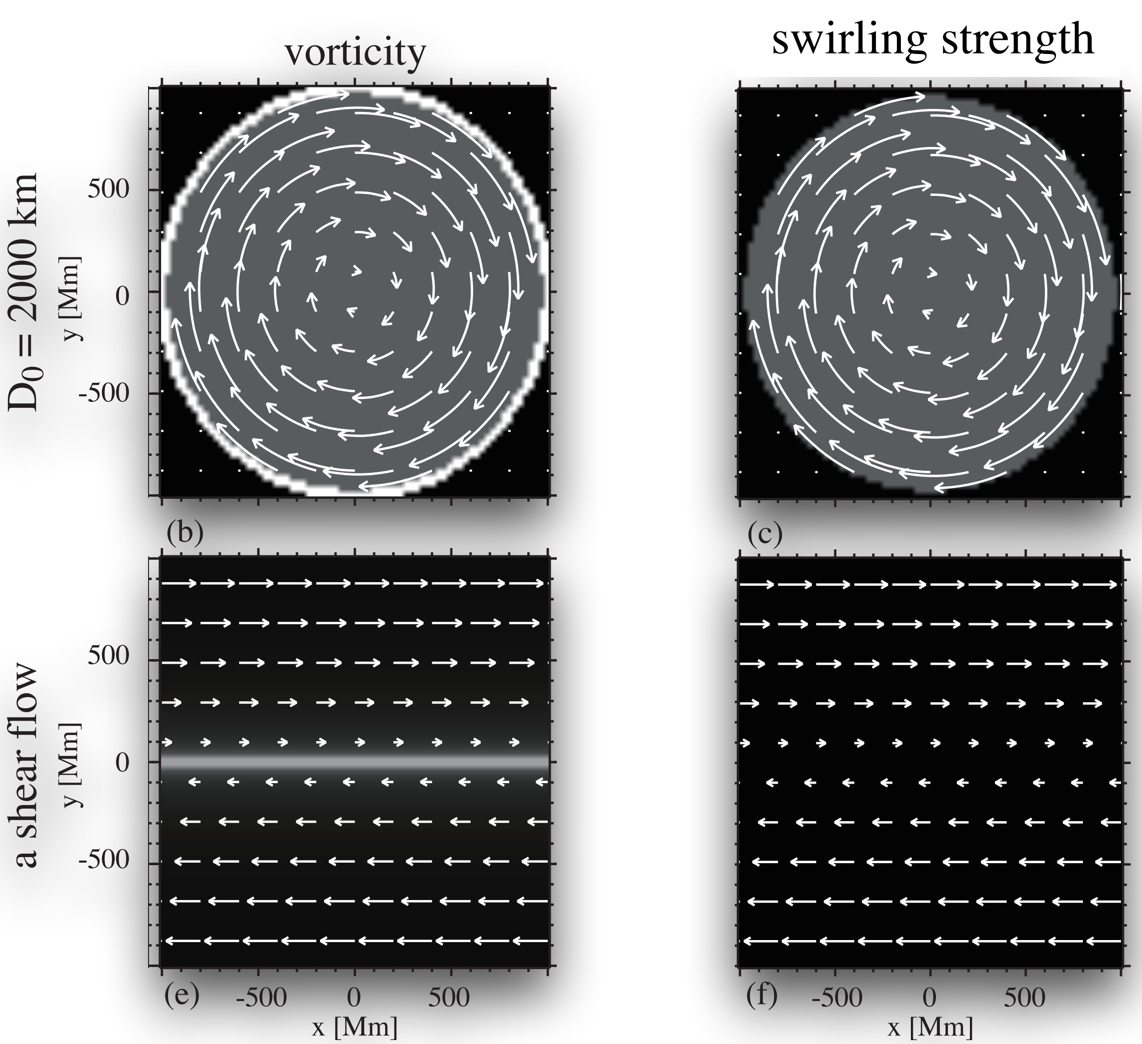
Hinode/Ca II H BFI movie
(courtesy by Mats Carlsson)

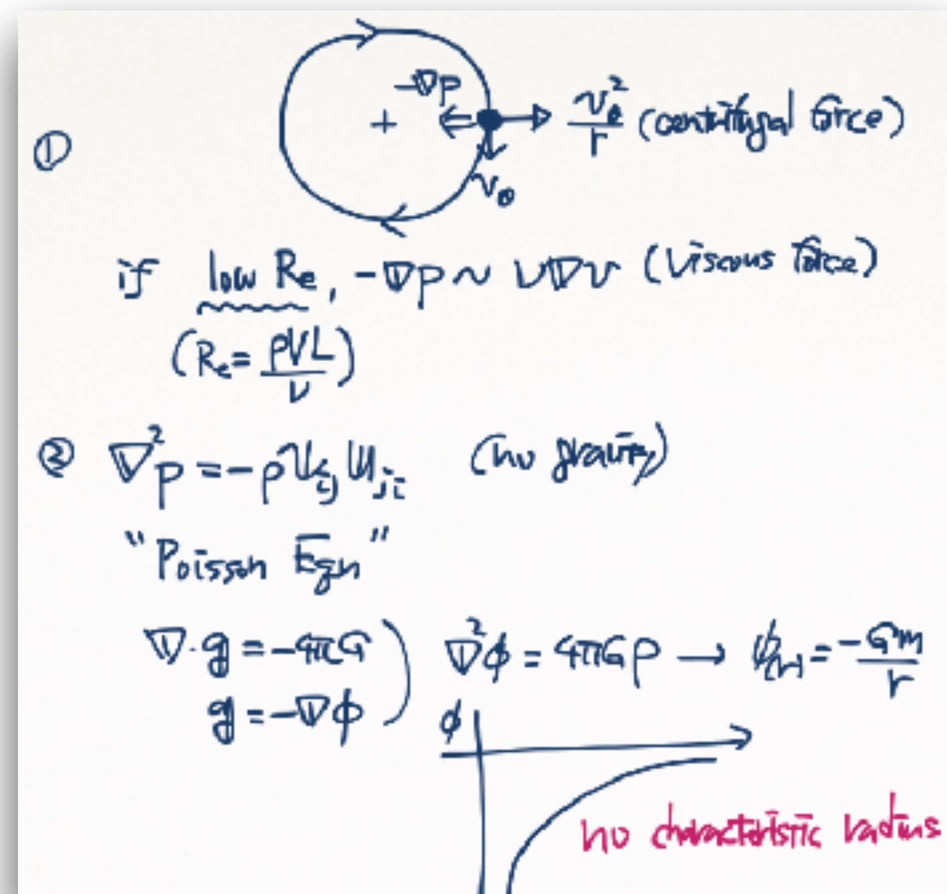
How to find a vortical structure?

- Streamlines
- Vorticity magnitude $\boldsymbol{\nabla} \times \boldsymbol{v} = \boldsymbol{\omega}$
- Local pressure minimum



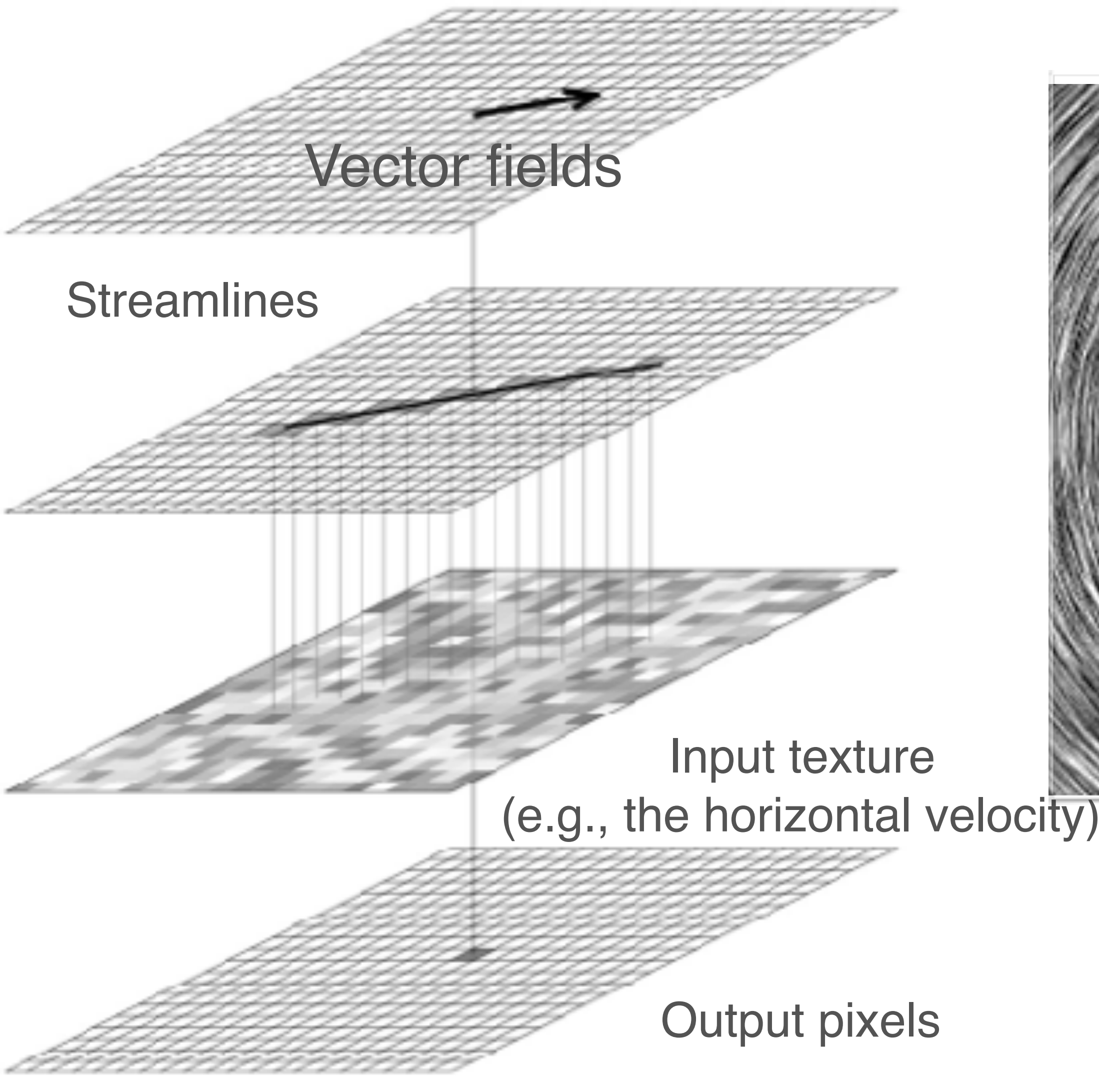
Rest frame | Slow co-moving frame | Fast co-moving frame





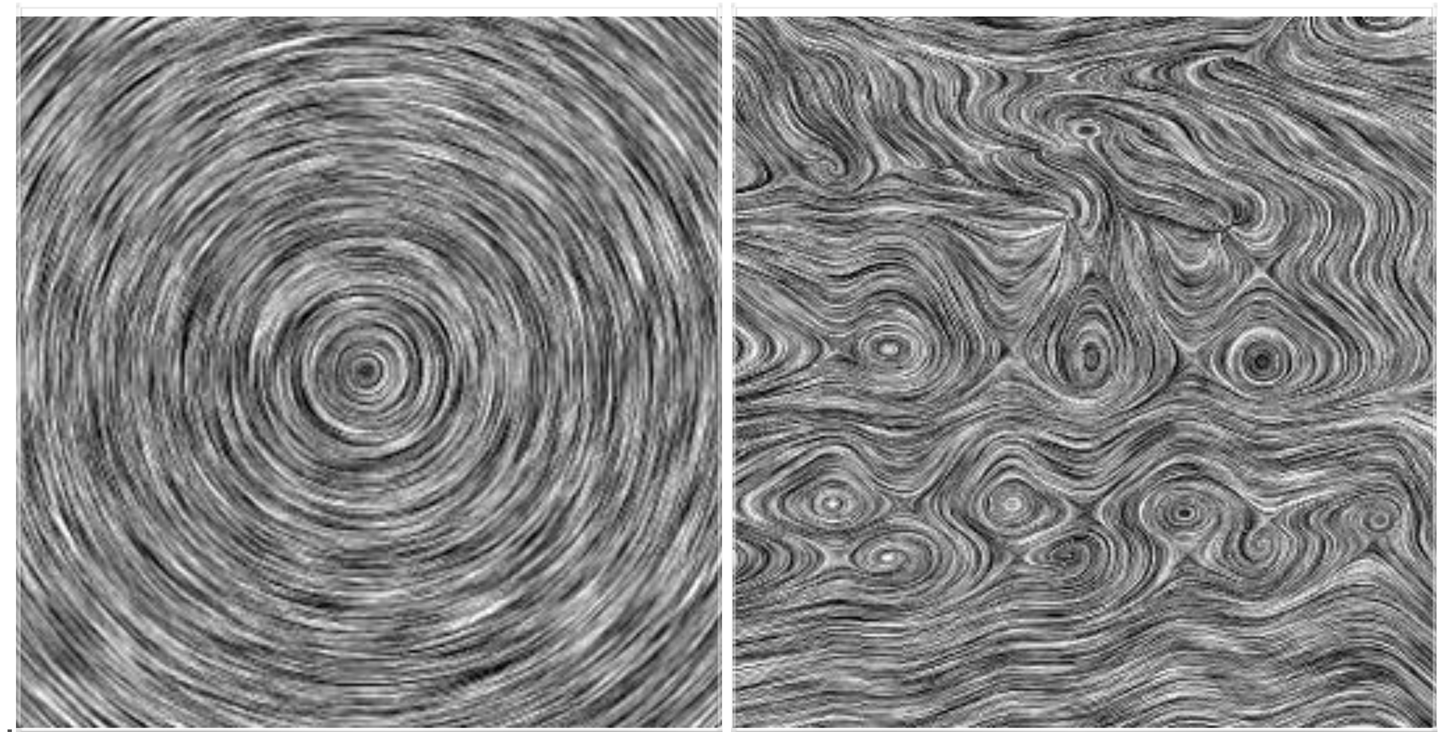
Fancy illustration of vortical structure

Line Integral Convolution (LIC)



Cabral & Leedom SIGGRAPH'93

Examples



Visualization of vortical flows
by using LIC imaging

What's a definition of a vortex?

J. Fluid Mech. (1995), *vol.* 285, *pp.* 69–94 69

On the identification of a vortex

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Department of Mechanical Engineering, University of Houston, Houston, TX 77204-4792, USA

(Received 6 December 1993 and in revised form 4 July 1994)

Considerable confusion surrounds the longstanding question of what constitutes a vortex, especially in a turbulent flow. This question, frequently misunderstood as academic, has recently acquired particular significance since coherent structures (CS) in turbulent flows are now commonly regarded as vortices. An objective definition of a vortex should permit the use of vortex dynamics concepts to educe CS, to explain formation and evolutionary dynamics of CS, to explore the role of CS in turbulence phenomena, and to develop viable turbulence models and control strategies for turbulence phenomena. We propose a definition of a vortex in an incompressible flow in terms of the eigenvalues of the symmetric tensor $\boldsymbol{S}^2+\boldsymbol{\Omega}^2$; here $\boldsymbol{S}$ and $\boldsymbol{\Omega}$ are respectively the symmetric and antisymmetric parts of the velocity gradient tensor $\nabla\boldsymbol{u}$. This definition captures the pressure minimum in a plane perpendicular to the vortex axis at high Reynolds numbers, and also accurately defines vortex cores at low Reynolds numbers, unlike a pressure-minimum criterion. We compare our definition with prior schemes/definitions using exact and numerical solutions of the Euler and Navier–Stokes equations for a variety of laminar and turbulent flows. In contrast to definitions based on the positive second invariant of $\nabla\boldsymbol{u}$ or the complex eigenvalues of $\nabla\boldsymbol{u}$, our definition accurately identifies the vortex core in flows where the vortex geometry is intuitively clear.

1. Introduction

The concept of vortices is as old as the subject of hydrodynamics; yet, an accepted definition of a *vortex* is still lacking. Turbulence is viewed as a tangle of vortex filaments, and much of turbulence physics is well explained using the concepts of vortex dynamics (e.g. see Tennekes & Lumley 1972; Hunt 1987). Turbulent shear flows have been found to be dominated by spatially coherent, temporally evolving vortical motions, popularly called *coherent structures* (CS) (Cantwell 1981; Lumley 1981; Hussain 1980). Vortex dynamics, which govern the evolution and interaction of CS and coupling of CS with background turbulence, is promising not only for understanding

1. Introduction

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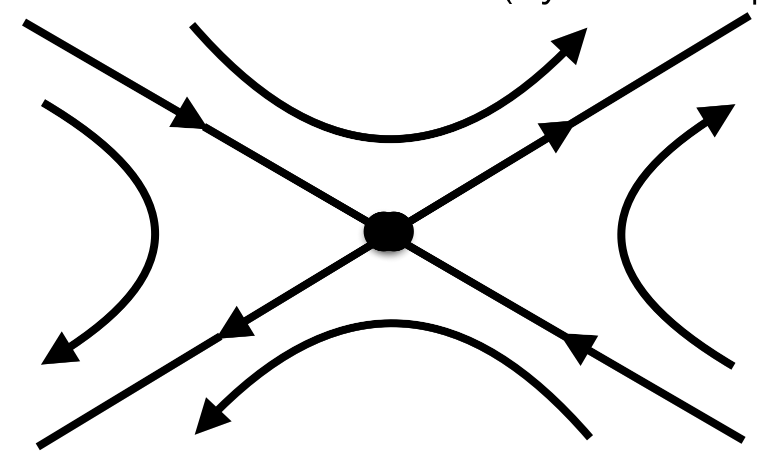
detectable regularity in time and space. Hence, a reference signal such as velocity can be used as a trigger for eduction (Hussain & Zaman 1980; Cantwell & Coles 1983).

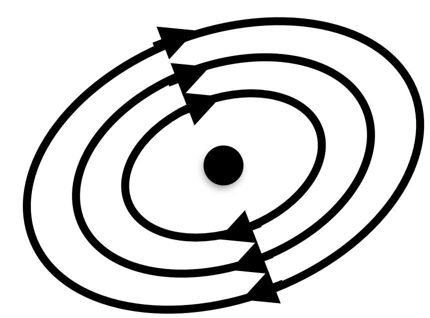
$$\nabla v = S + \Omega$$ the velocity gradient tensor

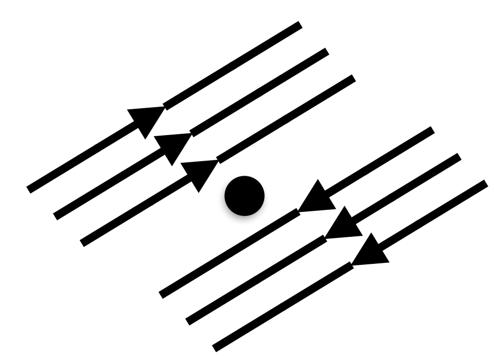
Galilean-invariants

$$S = \frac{1}{2}\left[\nabla v + (\nabla v)^{\mathrm{T}}\right]$$ the rate-of-strain tensor (symmetric part)



$$\Omega = \frac{1}{2}\left[\nabla v - (\nabla v)^{\mathrm{T}}\right]$$ the vorticity tensor (Anti-symmetric part)





Recent measures of a vortex

An imaginary part of eigenvalue of the velocity gradient tensor "Swirling Strength" (Moll et al. 2011)

Consider the velocity gradient tensor, $\mathcal{D}_{ij}$

$$\mathcal{D}_{ij} = \frac{\partial v_{\mathrm{i}}}{\partial x_{\mathrm{j}}} \tag{2}$$

If λ are the eigenvalues of $\mathcal{D}_{ij}$, then

$$\left[\mathcal{D}_{ij} - \lambda I\right] e = 0 \tag{3}$$

where e is the eigenvector.

The eigenvalues can be determined by solving the characteristic equation

$$\det\left[\mathcal{D}_{ij} - \lambda I\right] = 0 \tag{4}$$

which, for a velocity flow in two-dimensional space $\boldsymbol{v} = (v_{\mathrm{x}}, v_{\mathrm{y}})$, can be written as

$$\lambda^2 + P\lambda + Q = 0 \tag{5}$$

where $P = -\mathrm{tr}(\mathcal{D}_{ij})$ and $Q = \det(\mathcal{D}_{ij})$. Equation (5) has the following canonical solutions:

$$\lambda = \frac{-P \pm \sqrt{P^2 - 4Q}}{2}. \tag{6}$$

The critical point analysis/classification on the differential equations

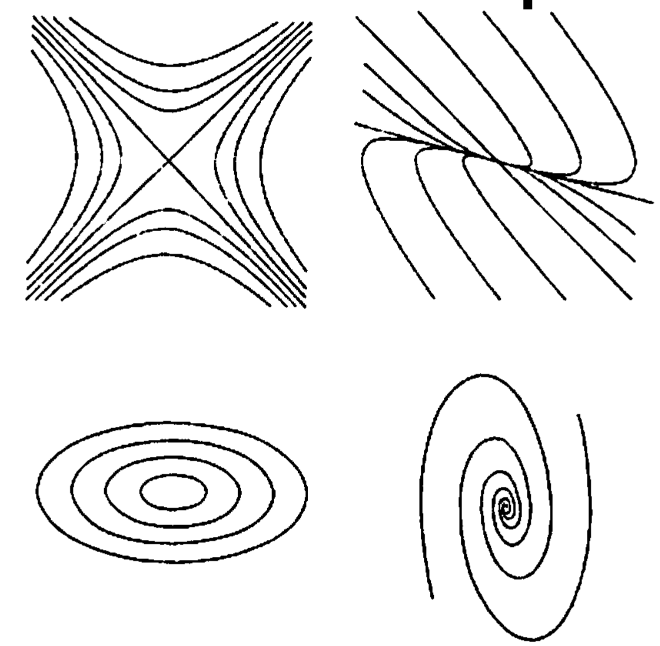


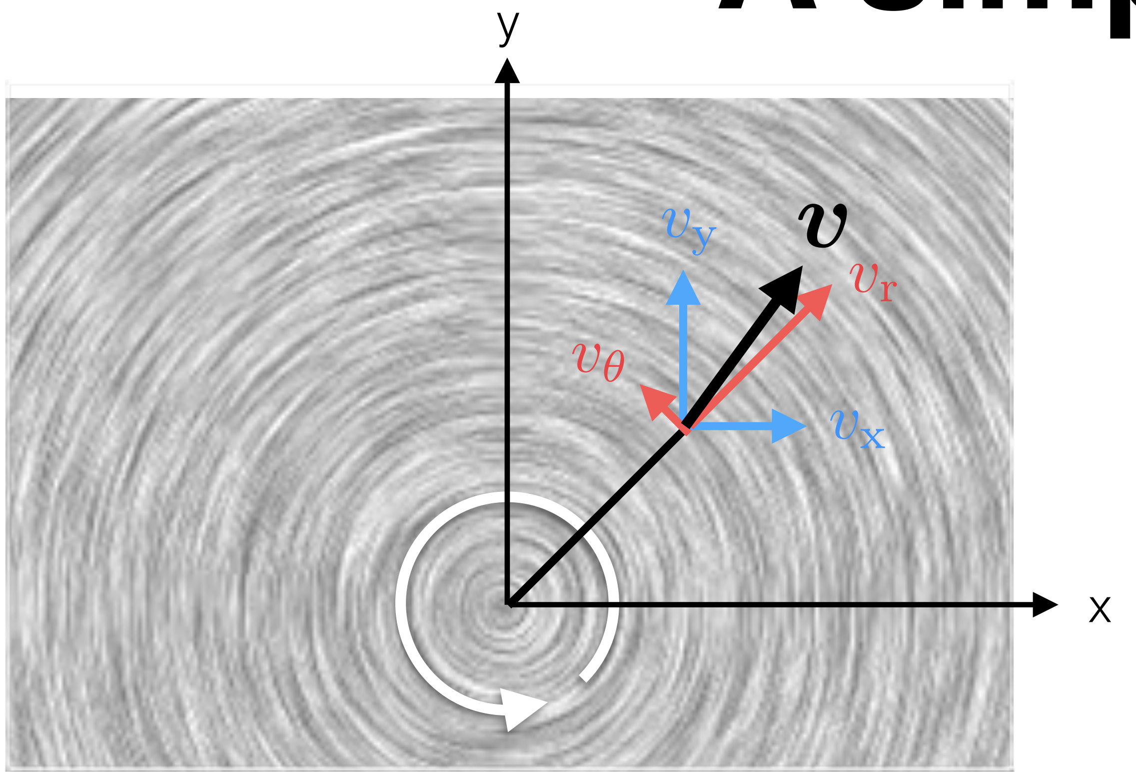
Figure 2.3
Types (topologies) of critical points. From top-left to bottom-right, types are saddle, node, center, and spiral.

Table 2.1 Roots and Types of Critical Points.

Roots	Type
Two real roots with opposite sign	saddle
Two real roots with same sign	node
Pure imaginary roots	center
Complex roots	spiral

"Black Hole Accretion Disks" 2008 p.66

A simple example



$$\begin{bmatrix} v_{\mathrm{x}} \\ v_{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_{\mathrm{r}} \\ v_\theta \end{bmatrix}$$

$$(v_{\mathrm{r}}, v_\theta) = (\mu, r\nu) \;\; \text{where } r = \sqrt{x^2 + y^2}$$

$$\boldsymbol{v} = \left[\mu\left(\frac{x}{r}\right) - \nu y\right]\boldsymbol{e}_{\mathrm{x}} + \left[\mu\left(\frac{y}{r}\right) + \nu x\right]\boldsymbol{e}_{\mathrm{y}}$$

$$\longrightarrow \mathcal{D}_{\mathrm{ij}} = \begin{bmatrix} \mu\left(\frac{x}{r^2}\right) & -\nu \\ \nu & \mu\left(\frac{y}{r^2}\right) \end{bmatrix}$$

$$\lambda^2 + P\lambda + Q = 0$$

$$P = -\mathrm{tr}(\mathcal{D}_{ij}) = \frac{(x+y)}{r^2}\mu$$

$$Q = \det(\mathcal{D}_{ij}) = \left(\frac{xy}{r^4}\right)\mu^2 + \nu^2$$

$$\lambda = \frac{-P \pm \sqrt{P^2 - 4Q}}{2}.$$

- a pure circular flow

$$\mu = 0,\;\; \nu \neq 0 \longrightarrow \;\; P = 0,\;\; Q = \nu^2$$

a conjugate pair of complex roots of $\lambda = \pm i\nu$

- a radial, non-circular flow

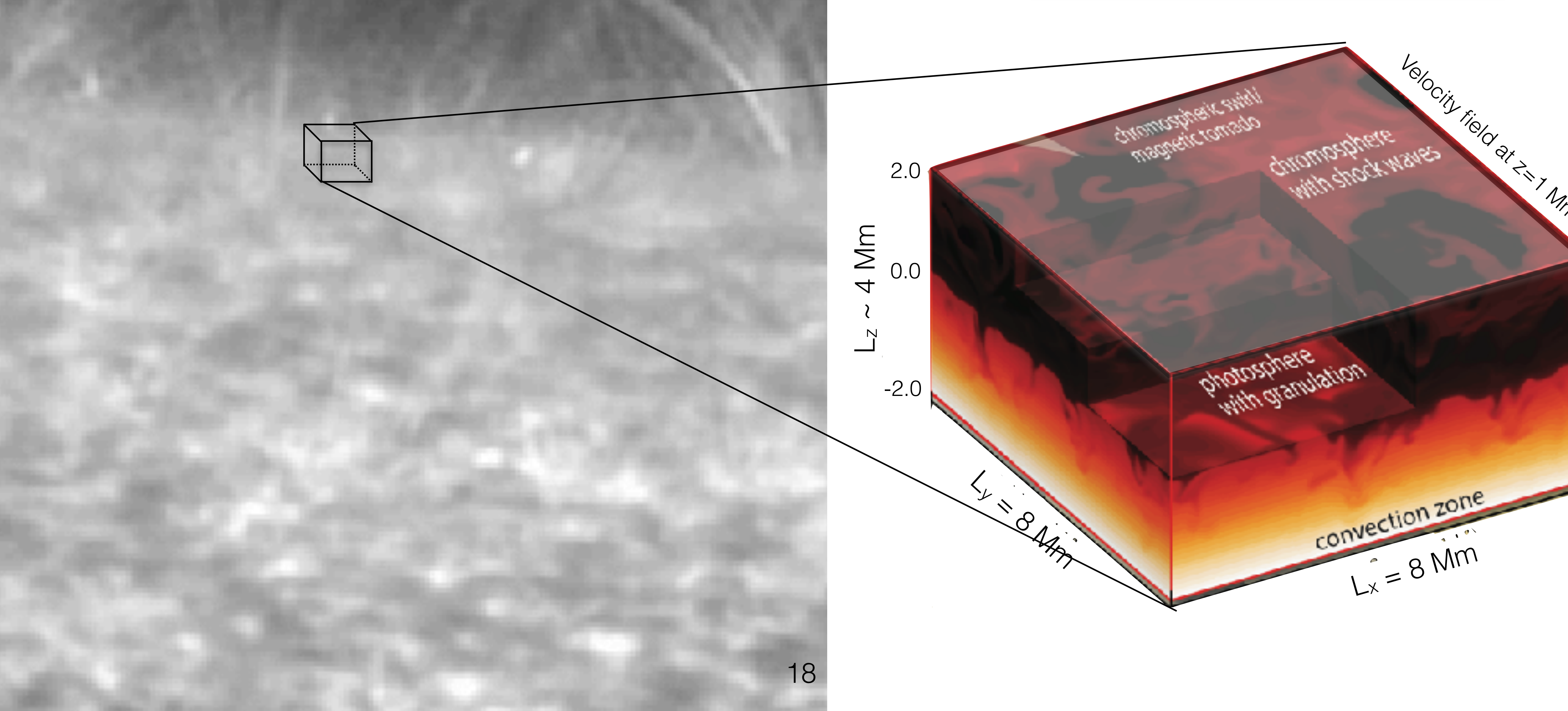
$$\mu \neq 0,\;\; \nu = 0 \longrightarrow \;\; P = -\frac{x+y}{r^2}\mu,\;\; Q = \frac{xy}{r^4}\mu^2$$

two distinctive real roots of $\lambda = \mu/r^2 x$ and $\mu/r^2 y$

Testing vortex detection algorithm

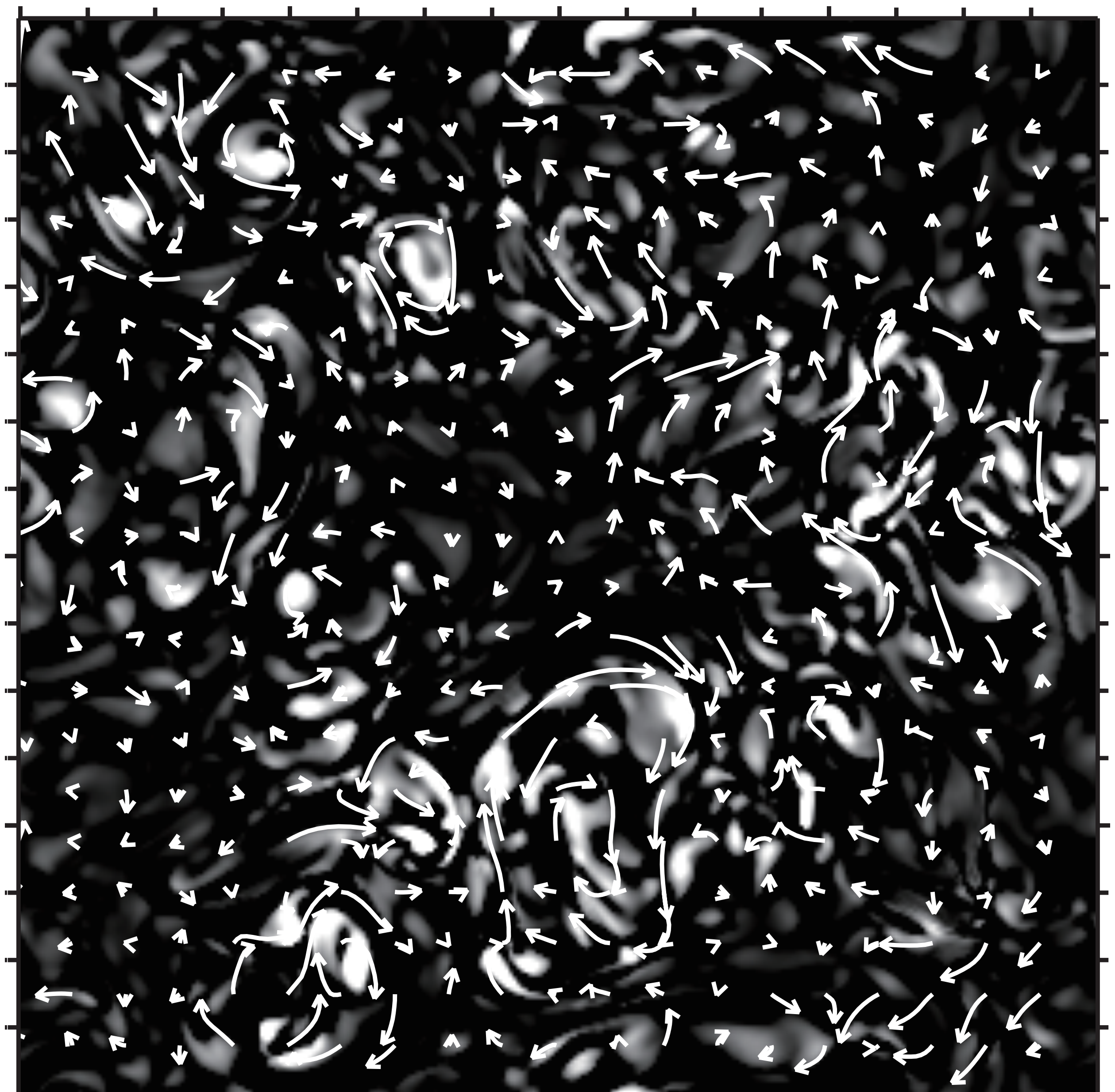
by using solar atmospheric models

The velocity field at z=1 Mm is used
as an input for testing
the detection algorithm

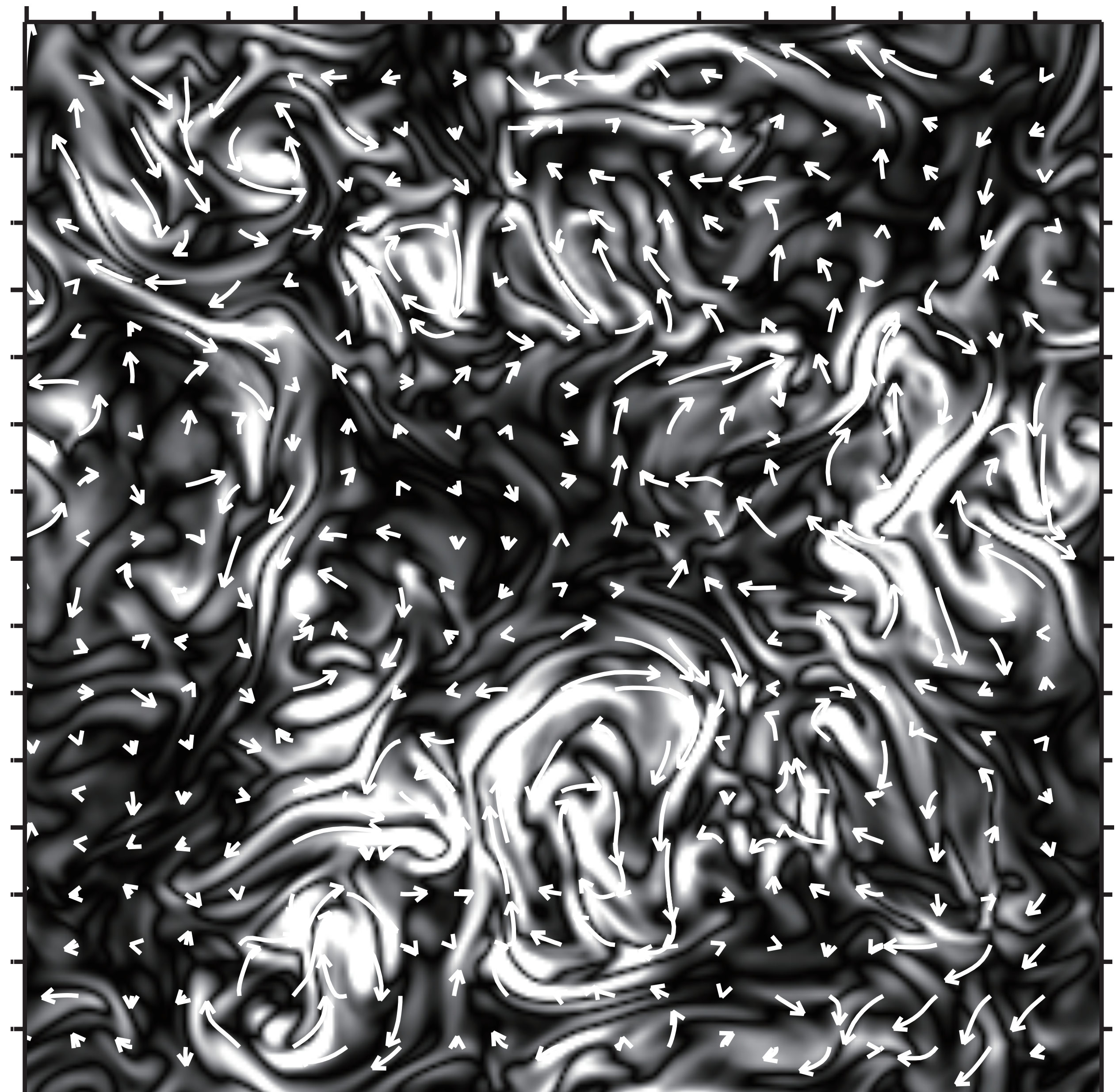


Snapshots of vortical structure

at the chromospheric height in the solar model atmosphere



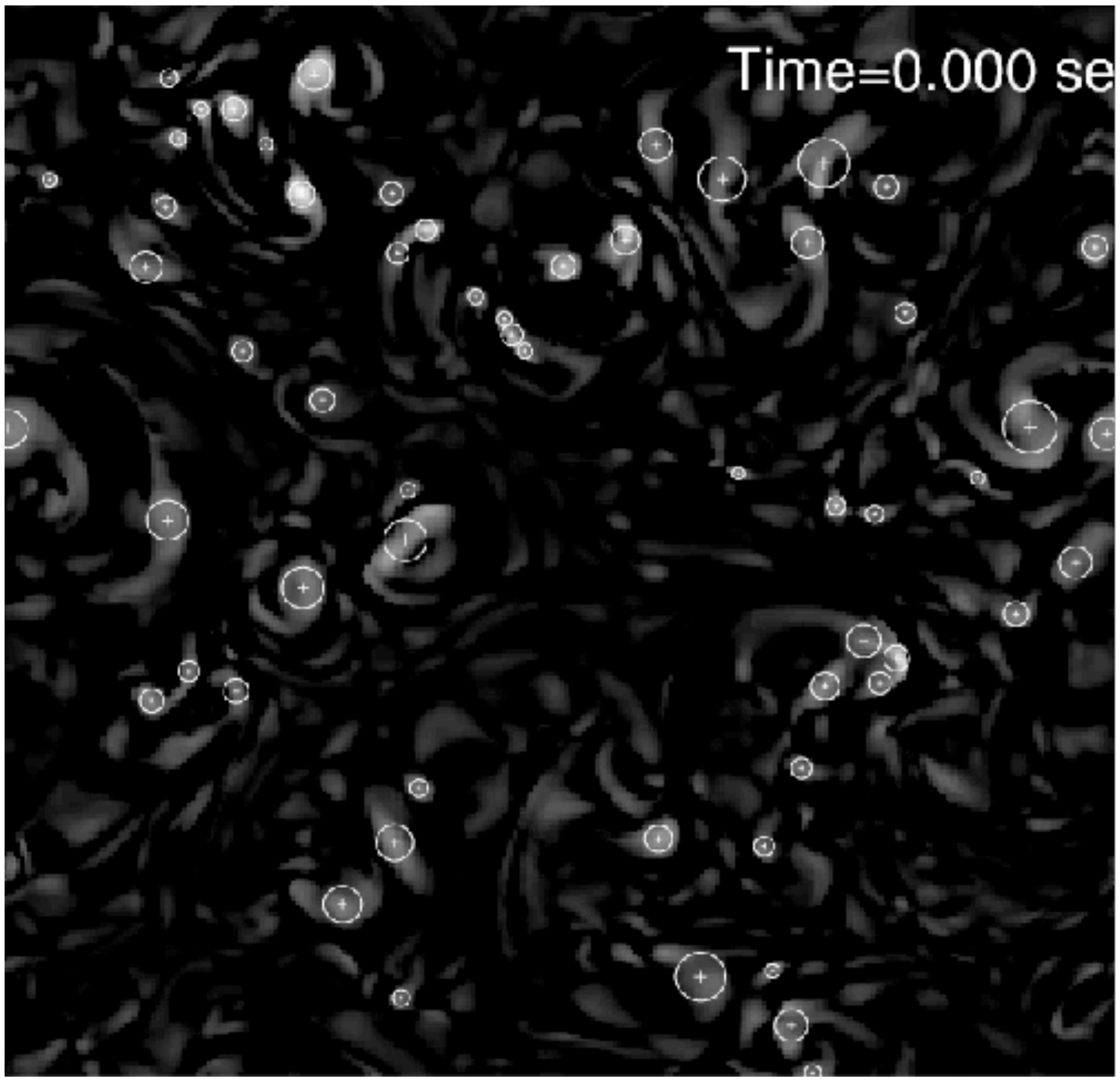
Swirling strength



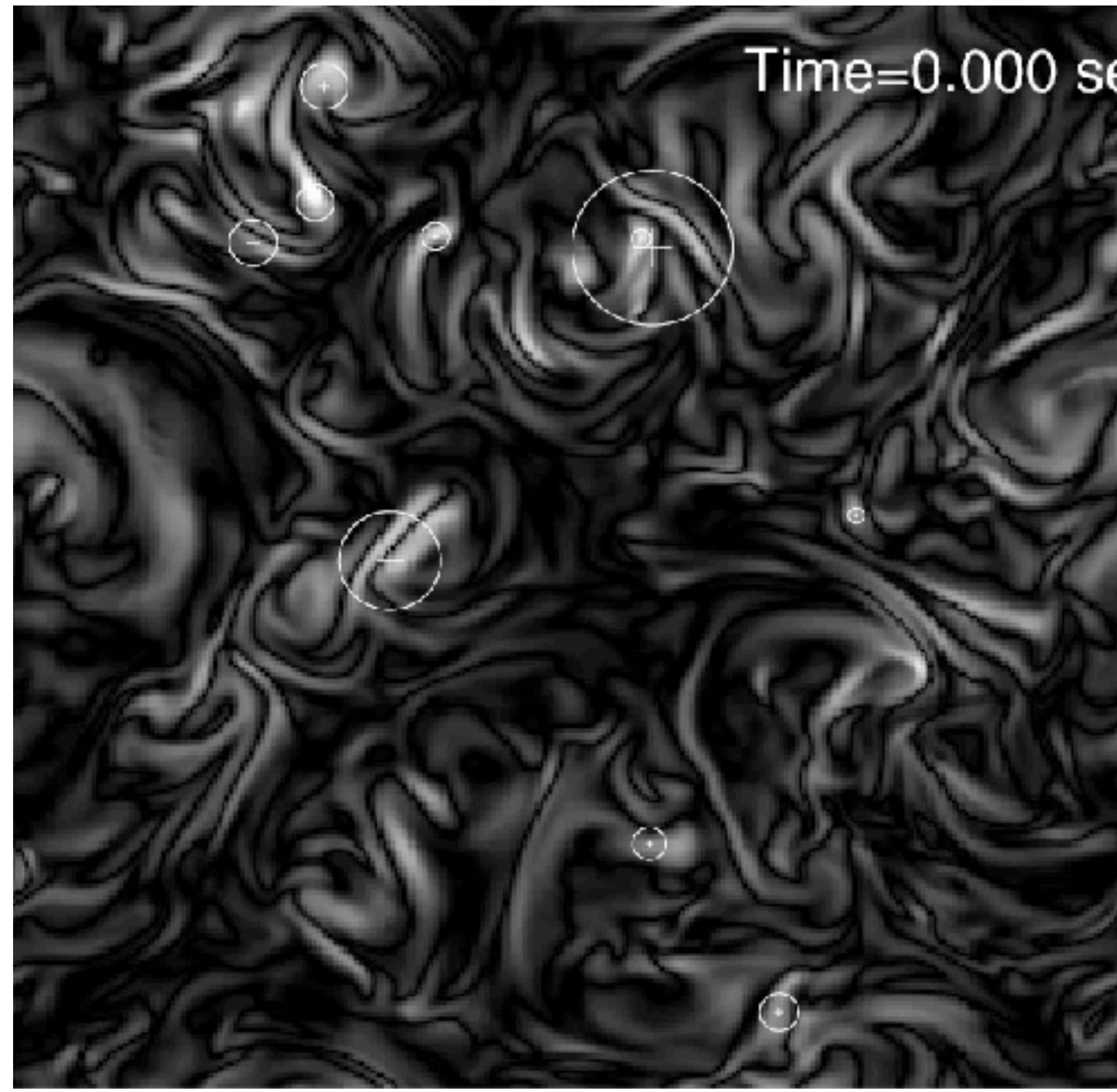
Vorticity

Comparison between swirling strength and vorticity

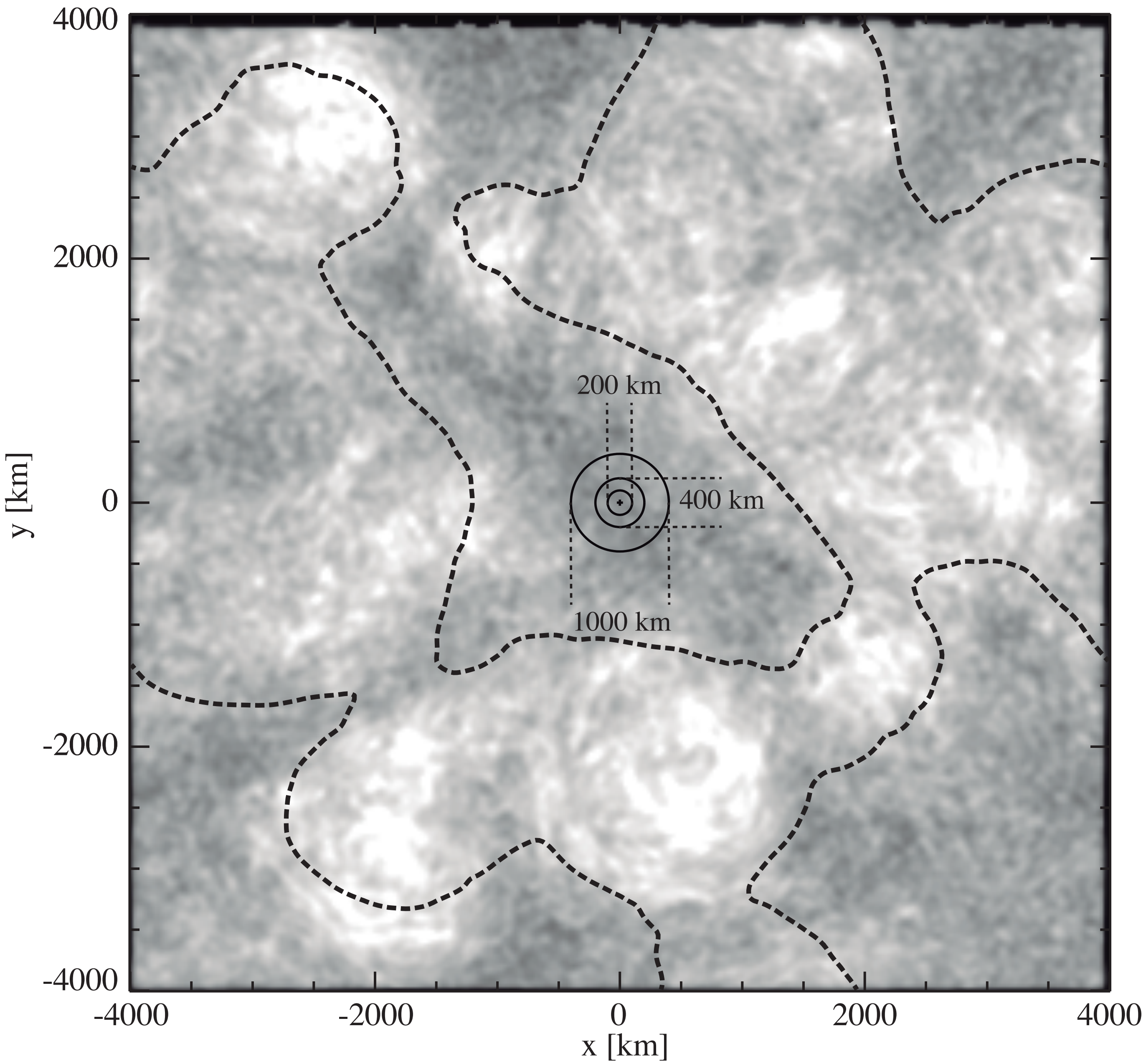
swirling strength



vorticity



Correlation between vortical structure and enhanced magnetic structure



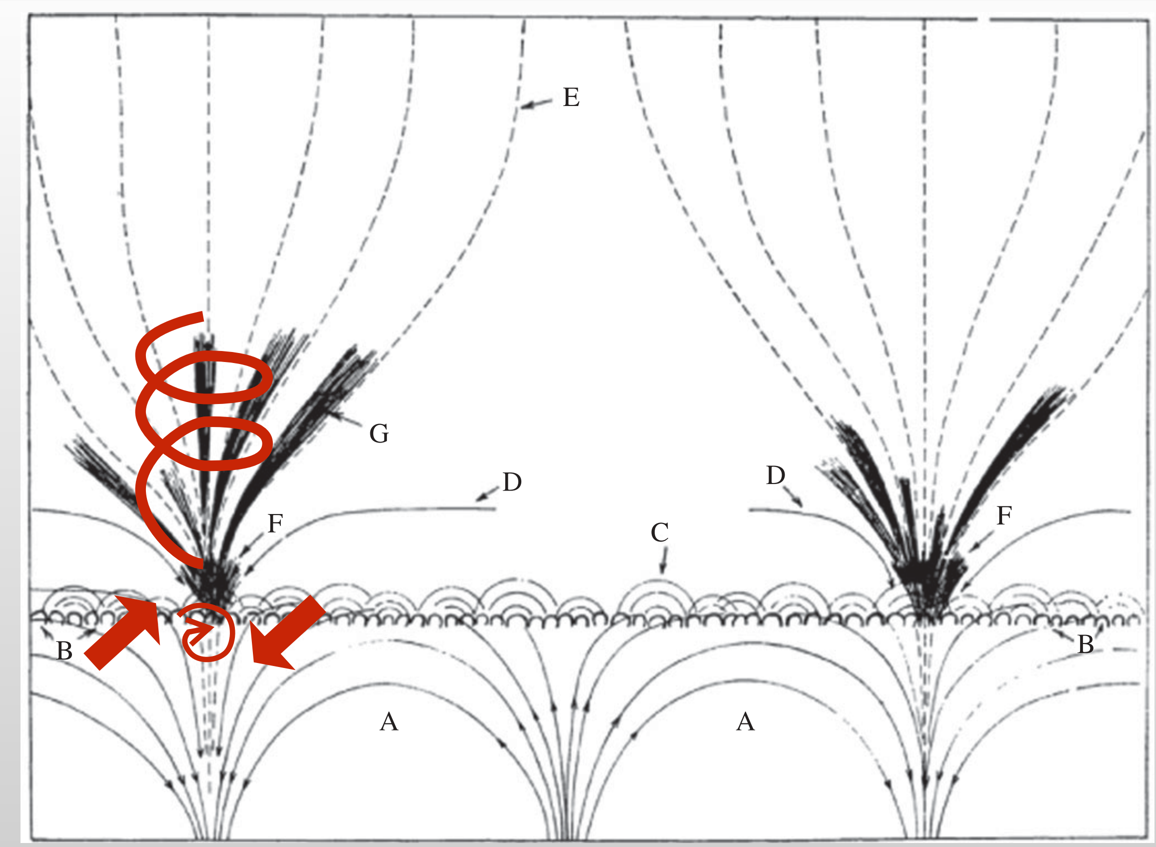
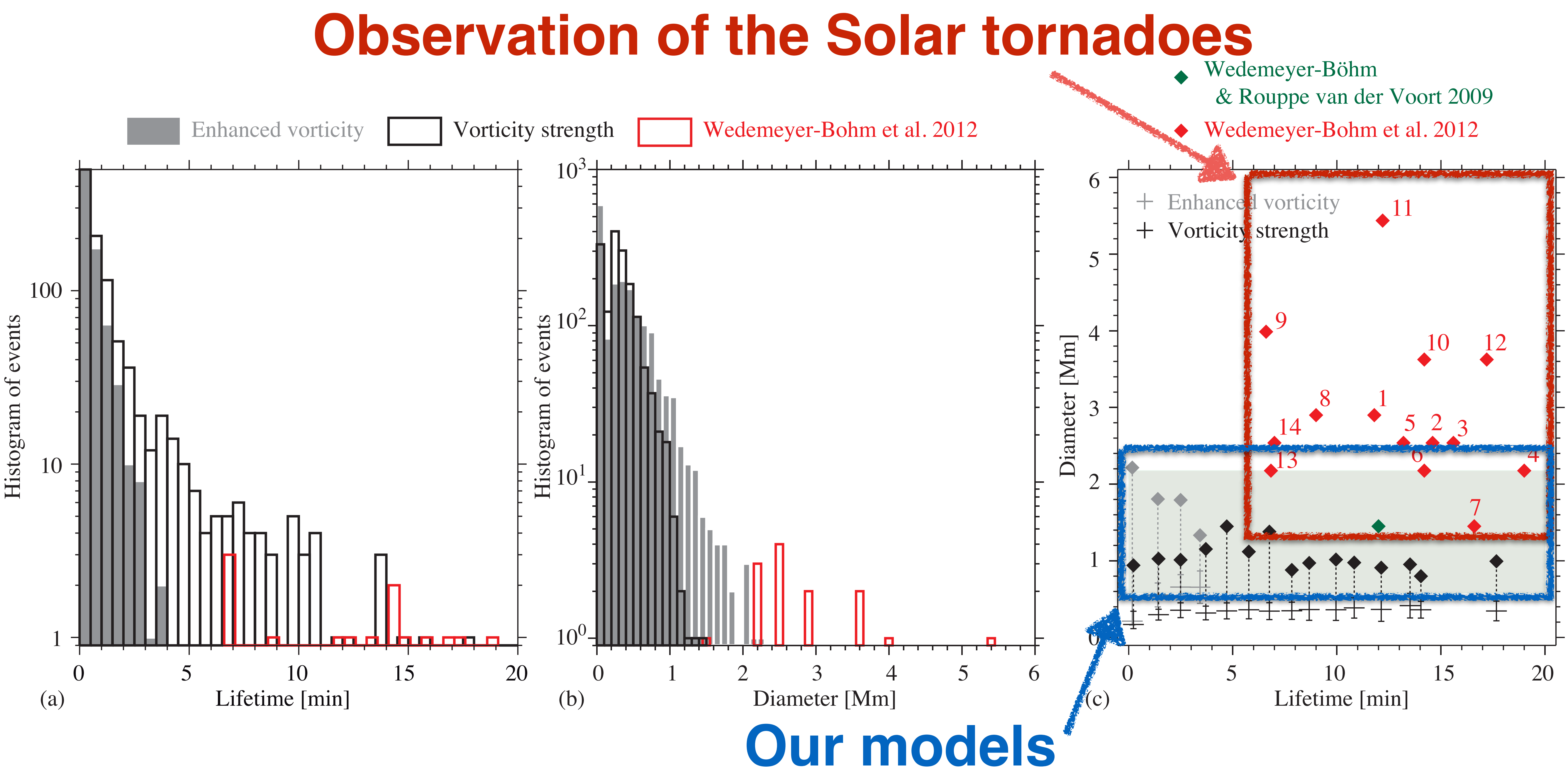


Figure 1. Sketch of the granulation–supergranulation–spicule complex in cross section. A, Flow lines of a supergranulation cell. B, Photospheric granules. C, Wave motions. D, Large-scale chromospheric flow field seen in Hα. E, [Magnetic] lines of force, pictured as uniform in the corona but concentrated at the boundaries of the supergranules in the photosphere and chromosphere. F, Base of a spicule 'bush' or 'rosette', visible as a region of enhanced emission in the Hα and K-line cores. G, Spicules. [. . .] The distance between the bushes is 30 000 km. Reproduced with permission from Noyes [15], including this caption.

Rutten 2012

Comparison with the Solar observations



Reference

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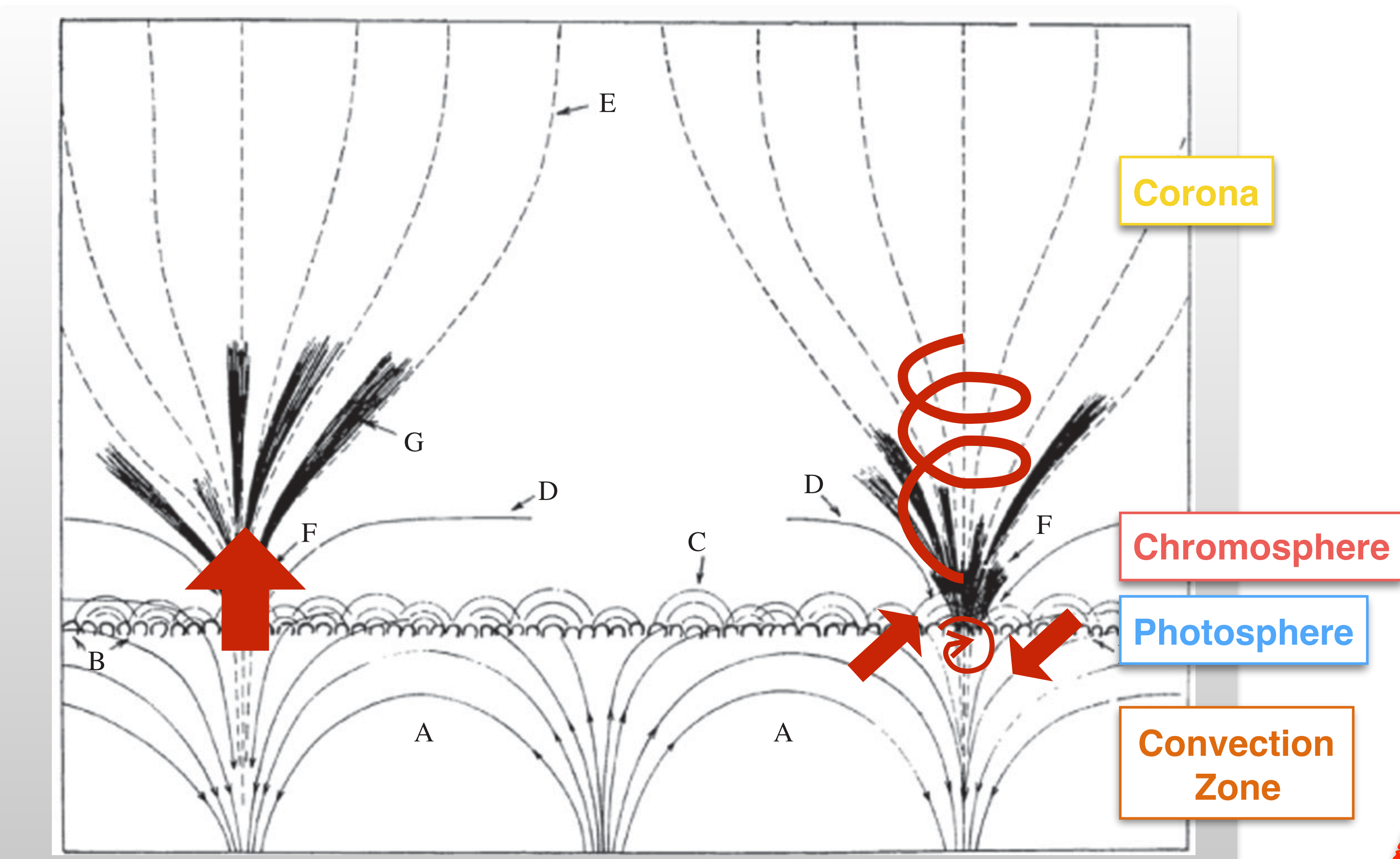
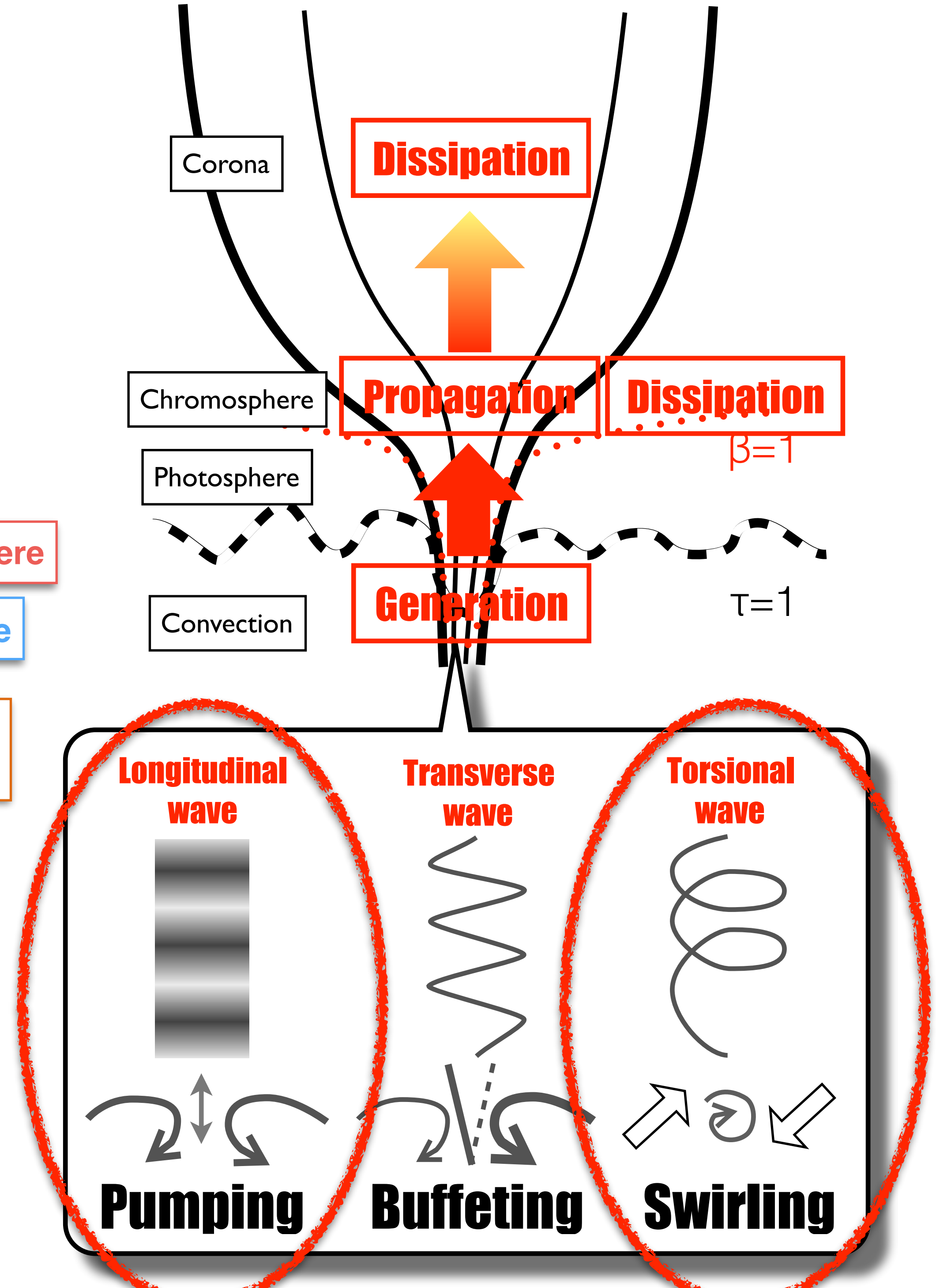
Summary



Rutten 2012



Magnetic flux sheath is a portal of energy transfer in the solar atmosphere