Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 8 November 2017 (MN LaTeX style file v2.2)

Single Star Implementation Notes

PFH

8 November 2017

1 METHODS

Our simulations use GIZMO (**?**),[1], and include full self-gravity, adaptive resolution, ideal and non-ideal magneto-hydrodynamics (MHD), detailed cooling and heating physics, protostar formation, accretion, and feedback in the form of protostellar jets and radiative heating, and main-sequence stellar feedback in the form of photo-ionization and photo-electric heating, radiation pressure, stellar winds, and supernovae. A subset of our runs include explicit treatment of the dust particle and cosmic ray dynamics as well as radiation-hydrodynamics; otherwise these are included in simplified form.

1.1 Gravity

All our simulations include full self-gravity for gas, stars, and dust. The gravity solver in GIZMO is a heavily modified version of the Tree-PM method in GADGET-3 (**?**). Because this is a Lagrangian code, there is no fixed "spatial resolution" for gas: gravitational force softenings scale adaptively (in a fully-conservative manner; see **?**) with the inter-particle separation and can (in principle) become arbitrarily small (such that the assumed gas distribution determining the gravitational forces at the resolution limit is identical to the assumption in the hydrodynamic equations). Softenings for star particles are set sufficiently small that they are always Keplerian, though we find no difference setting them adaptively based on a "nearest neighbors" distance to surrounding gas particles.

1.2 Fluid Dynamics

Our MHD simulations use the Lagrangian finite-volume "meshless finite mass" Godunov method in GIZMO, which captures advantages of both grid-based and smoothed-particle hydrodynamics (SPH) methods. In **????** we consider extensive surveys of test problems in both hydrodynamics and MHD, and demonstrate accuracy and convergence in good agreement with well-studied state-of-the-art regular-mesh finite-volume Godunov methods and moving-mesh codes (e.g. ATHENA & AREPO; **??**), especially for super-sonic and sub-sonic MHD turbulence and instabilities such as the Hall MRI.

1.2.1 Turbulence

To reflect the turbulent formation and environment of realistic clouds, we drive turbulence on the largest (box-size) scales in the simulation. The driving follows **?**, using the method from **???**: a small range of modes (wavelengths $1/2-1$ times the cloud diameter $L_{\rm cloud}$) are driven as an Ornstein-Uhlenbeck process in Fourier space, with the compressive part of the modes projected out via Helmholtz decomposition so that we can specify the ratio of compressible and incompressible/solenoidal modes. Unless otherwise specified we adopt pure solenoidal driving, appropriate for e.g. galactic shear (so that we do not artificially "force" compression/collapse), but we vary this. The specific implementation here has been verified in **????**.

The driving specifies the large-scale steady-state (one-dimensional) turbulent velocity $\sigma_{1D} \equiv \langle v_{\rm turb,\,1D}^2\rangle^{1/2}$ and Mach number $\mathcal{M} \equiv \langle (v_{\rm turb,\,1D}/c_s)^2\rangle^{1/2}$, where $v_{\rm turb,\,1D}$ is an (arbitrary) projection (in detail we average over all random projections). Unless otherwise specified, we initialize our clouds on the linewidth-size relation from **?**, with $\sigma_{1D} \approx 0.7\,{\rm km\,s^{-1}}\,(L_{\rm cloud}/{\rm pc})^{1/2}$

1.2.2 Magnetic Fields

Unless otherwise specified all simulations include magnetic fields. An initially constant mean-field $\mathbf{B}_0$ is evolved with the turbulence so that runs begin with fully-developed turbulent field structure. By default, we take $|\mathbf{B}_0| = 10\,\mu G\,n_4^{2/3}$ where $n_4 = \langle n_0/10^4\,{\rm cm^{-3}}\rangle$ refers to the initial density of the cloud (n_0); the local fields quickly grow and saturate with amplitudes $\langle|\mathbf{B}|^2\rangle^{1/2} \sim 60\,\mu G\,n_4^{2/3}$. We consider variations in the mean field strength.

1.2.3 Non-Ideal Effects

By default, we solve the ideal MHD equations, but (optionally) include various non-ideal processes. These include:

(i) **Non-ideal MHD:** Ambipolar diffusion, Ohmic resistivity, and the Hall effect: in dense, neutral gas non-ideal MHD effects can be important. All three effects are always integrated explicitly and treated self-consistently (for all gas elements), with coefficients calculated on-the-fly independently for all gas elements based on their temperature, density, magnetic field strength, and ionization states of all tracked ions and dust grains (accounting for the full grain size spectrum, variable local dust-to-gas ratios, and locally variable abundance patterns, and the details of the followed local radiation and cosmic ray fields). For more details see § A1.

(ii) **Spitzer-Braginskii conduction & viscosity:** In hot gas (generated by shocks and feedback), conduction and viscosity can be important. We solve the fully anisotropic conduction and viscosity equations as described in **?**, using the standard (temperature, density, and ionization-state dependent) Spitzer-Braginskii conduction and viscosity coefficients as detailed in **?**. For details see § A2.

(iii) **Passive scalar (metal) diffusion:** Because our default hydrodynamic method follows finite-mass elements,

[1] A public version of this code is available at `http://www.tapir.caltech.edu/~phopkins/Site/GIZMO.html`.

micro-physical diffusion of passive scalars (e.g. metals) in a turbulent ISM requires an explicit numerical diffusion term. This is implemented following the standard diffusion operators in **?**; for details see § A3.

Extensive tests of the numerical implementations of each of these terms are presented in **?**. We showed there that the methods are higher-order accurate, fully conservative, competitive with state-of-the-art fixed-grid methods (in e.g. ATHENA), and can correctly capture the linear and non-linear behavior of anisotropic-diffusion-driven instabilities such as the magneto-thermal, heat-flux bouyancy-driven, and Hall-magnetorotational instabilities. In contrast, historical implementations of these physics in SPH methods tend to artificially suppress these instabilities (or can be numerically unstable).

1.2.4 Dust & Metals

Our simulations separately track 11 metal species as described above for the cooling physics. We assume initially uniform abundances, with total metal abundance Z and solar abundance ratios. Enrichment from stellar winds and supernovae, when they occur, follows (**?**): the appropriate metal yields tabulated from **?** are injected into the exact same gas elements surrounding the star as the corresponding mass, momentum, and energy. Unresolved metal diffusion between gas elements can be optionally modeled as described in § 1.2.3.

By default, we assume a constant dust-to-metals ratio, giving dust-to-gas mass ratio $f_{dg} = 0.5\,Z$, for computing all dust-based quantities.

However, in a subset of runs, we explicitly model the dust dynamics following **?**. ???

1.2.5 Cosmic Rays

By default, we assume a uniform cosmic ray ionization rate $\zeta = 10^{-17}\,\mathrm{s}^{-1}$, which we can freely vary.

In a subset of runs, we explicitly model the cosmic ray dynamics. ???

1.2.6 Radiation

???

1.3 Cooling & Heating Physics

The cooling physics here has been described in several previous papers (**?**). Gas heating & cooling is solved following a fully-implicit algorithm, as in **?**. Heating/cooling rates are computed explicitly including: free-free, bound-bound, photo-ionization, recombination, Compton, photo-electric, metal-line, fine-structure, molecular, dust-gas collisional, cosmic ray, and hydrodynamic (shocks/compression/expansion) processes. We separately track 11 species (H, He, C, N, O, Ne, Mg, Si, S, Ca, Fe) as well as dust and cosmic rays.

Ionization states and molecular fractions are computed from CLOUDY simulations assuming collisional+photo-ionization equilibrium, including the effects of a uniform meta-galactic background (from **?**) together with local radiation sources (see feedback, below). We account for self-shielding with a local Sobolev/Jeans-length approximation (integrating the local density at a given particle out to a Jeans length to determine a surface density Σ, then attenuating the flux seen at that point by $\exp(-\kappa_\nu\,\Sigma)$); this approximation has been calibrated in full radiative transfer experiments in **?** and **?**. At high densities the ionization is dominated by cosmic rays: we therefore also self-consistently compute the ionization state and grain charges following **?** (as described in § A1) for a mostly-neutral gas with a mixture of electrons, ions, neutrals, cosmic rays, and dust grains, and adopt this as our default when it yields a larger free electron fraction.

High-temperature ($> 10^4\,K$) metal-line excitation, ionization, and recombination rates then follow **?**. Free-free, bound-free, and bound-bound collisional and radiative rates for H and He follow **?** with the updated fitting functions in **?**. Photo-electric rates follow **?**, accounting for PAHs and local variations in the dust abundance. Compton heating/cooling (off the combination of the CMB and local sources) follows **?** (allowing in principle for two-temperature plasma effects, although these are not important here). Fine-structure and molecular cooling at low temperatures ($5-10^4\,\mathrm{K}$) follows a pre-computed tabulation of CLOUDY runs as a function of density, temperature, metallicity, and local radiation background (see **?**). Collisional dust heating/cooling rates follow **?** with updated coefficients from **?** assuming a minimum grain size of $10\,\text{Å}$, and dust temperatures calculated below. Cosmic ray heating follows **?** accounting for both hadronic and Compton interactions, with the cosmic ray background estimated as described below. Hydrodynamic heating/cooling rates (including shocks, adiabatic work, reconnection, resistivity, etc.) are computed in standard fashion by the MHD solver; this is incorporated directly into our fully-implicit solution rather than being operator-split, since operator-splitting can lead to large errors in temperature in the limit where the cooling time is much faster than the dynamical time.

At sufficiently high densities, gas becomes optically thick to its own cooling radiation. In simulations with explicit multi-frequency radiative transfer, described below, this can be handled explicitly. In our default simulations, however, we can accurately approximate the optically-thick cooling limit following **?** (the relevant approximations are checked against exact results for proto-planetary disks in **???**).[2]

[2] Following **?**, each gas element is treated as a midplane element embedded in an optically thick slab with surface density $\Sigma_{\rm slab}$ calculated according to our previous Sobolev approximation. We assume it is in LTE, which relates the temperature change at midplane to surface radiation by way of the optical depth, using the equations in Appendix A therein. To good approximation, the heating/cooling rate per element is "capped" at the maximum value $|dE/dt| = \sigma\,T_{\rm mid}^4\,(\Sigma_{\rm slab}/\mu)^{-1}/(1+\kappa_R\,\Sigma_{\rm slab})$, where $T = T_{\rm mid}$ is the element or "midplane" temperature, μ the mean molecular weight, and κ_R the Rosseland mean opacity. We calculate κ_R at low temperatures from the tables in **?**; at high temperatures ($> 1500\,\mathrm{K}$) we explicitly tabulate the Thompson, molecular-line, H^- ion, Kramers, and e^- conductivity terms as in **?**.

A 5 K temperature floor is enforced, although this has no detectable effect on our conclusions.

1.4 "Protostellar" Sink Particles

Dense, self-gravitating gas can collapse below our resolution limit, at which point it is treated via a standard sink-particle approach; these sink particles represent accreting sites of protostar formation and so act, in turn, back on the medium from which they formed.

1.4.1 Sink Formation

Following **?**, a gas resolution element (cell or particle) is converted to a sink particle when it meets the following criteria:

(i) **Self-gravitating:** We require the cell is locally self-gravitating. Specifically, ???
(ii) **Local extremum:** We only allow sink formation if the particle is a local density maximum among its ~ 32 nearest neighbors. As shown in **?**, this prevents spurious formation of multiple sink particles in a single resolution element that should collapse to form one object. We have alternatively considered requiring a local potential minimum, but this adds considerable computational expense and gives indistinguishable results in our tests.
(iii) **High-density:** The gas must exceed a minimum density $n > n_{\rm threshold}$. We typically adopt $n_{\rm threshold} = 100\,n_0$.
(iv) **Self-shielding:** The gas must be shielded from ambient UV radiation so it can cool. We adopt the column-density and metallicity-dependent threshold from **?**; this is trivially satisfied for almost all gas that meets our density and self-gravity criteria.
(v) **Jeans unstable:** We require a Jeans mass ???
(vi) **Converging flow:** The local velocity divergence (centered on the element) must satisfy $\nabla\cdot\mathbf{v} < 0$. This is usually satisfied given the self-gravity criterion above but prevents spurious sink formation in transient events.
(vii) **No duplication:** Sink formation is prohibited if there is another sink particle within the resolution element (i.e. within the radius including the nearest ~ 32 gas elements).

If all of these criteria are met, a gas particle is instantly converted into a sink particle, conserving mass.

1.4.2 Sink Growth

Once formed, sinks can grow via two accretion channels:

(i) **Resolved Gravitational Capture:** If a gas element b is explicitly resolved and bound to the sink, it is immediately captured by the protostar. We require (1) that the element have velocity relative to the sink below the Keplerian escape velocity $|\mathbf{v}_b - \mathbf{v}_{\rm sink}| < v_{\rm esc}(M_{\rm sink},\,r_b)$ where $r_b \equiv |\mathbf{x}_b - \mathbf{x}_{\rm sink}|$; (2) that it be bound to the sink plus local gas enclosed if we spread the mass in the particle such that it forms an isothermal sphere around the sink: $E_{{\rm thermal},\,b} + E_{B,\,b} + (m_b/2)\,|\mathbf{v}_b - \mathbf{v}_{\rm sink}|^2 < 2\,G\,M_{\rm sink}/r_b + 4\pi\,G\,\rho_b\,r_b^2$; and (3) that the apocentric radius of the orbit of b (assuming a Keplerian test-particle orbit about the sink with the given r_b and $|\mathbf{v}_b - \mathbf{v}_{\rm sink}|$) is less than twice the resolution scale (the kernel search radius around the sink). If a gas element b meets this criteria for more than one sink particle it is accreted onto whichever dominates the local acceleration (i.e. larger value of $M_{\rm sink}/|\mathbf{x}_b - \mathbf{x}_{\rm sink}|^2$).

(ii) **Diffuse Accretion:** If e.g. the initial protostellar mass is small, it may be impossible to resolve accretion via (i) above. We therefore estimate the accretion radius:

$$R_{\rm acc} \equiv \frac{G\,M_{\rm sink}}{c_s^2 + v_{\rm A}^2 + \langle|\mathbf{v}_{\rm gas} - \mathbf{v}_{\rm sink}|^2\rangle} \tag{1}$$

where the surrounding gas properties (c_s, ρ, $\langle|\mathbf{v}_{\rm gas}-\mathbf{v}_{\rm sink}|^2\rangle$) are computed via a kernel-weighted mean from the gas particles surrounding the sink. If the gas is smooth on sub-resolution scales, then that within $R_{\rm acc}$ is bound; so if $R_{\rm acc}$ is unresolved (smaller than the kernel search radius), we add a continuous mass accretion following Bondi-Hoyle-Lyttleton theory:

$$\dot{M}_{BHL} \equiv 4\pi\,\rho\,R_{\rm acc}^2\,v_{\rm ff}(R_{\rm acc}) \tag{2}$$

The sink grows continuously by $\dot{M}_{\rm BHL}\,\Delta t$ in a timestep Δt, algorithmically implemented following **?**. Note that if $R_{\rm racc}$ is resolved, this is in the regime where our "resolved gravitational capture" method should correctly identify accretion, so we set $\dot{M}_{BHL} = 0$.
(iii) **Sink Mergers:** ???

From Gravitational Capture to the Protostar: The above criteria (i)-(ii) govern gravitational capture of gas. Once captured, however, this is not immediately accreted onto a protostar. We therefore divide the sink mass into two components: $M_{\rm sink} = M_{\rm ps} + M_{\rm disk}$, a "protostar" mass $M_{\rm ps}$ and a "reservoir" mass $M_{\rm disk}$ (which represents some combination of accretion disk and unresolved collapsing core). At sink formation, $M_{\rm ps} = 0$, and whenever $M_{\rm sink}$ increases, the mass is added to $M_{\rm disk}$. We therefore require some prescription to transfer mass from $M_{\rm disk}$ to $M_{\rm ps}$. We have considered (i) instantaneous transfer every timestep (no "reservoir"), (ii) constant $\dot{M}_{\rm ps} = c_s^3/G$, as long as $M_{\rm disk} > 0$, (iii) constant disk depletion rate $\dot{M}_{\rm ps} = M_{\rm disk}/t_{\rm dep}$ with $t_{\rm dep} \sim 10^5\,{\rm yr}$, (iv) infall at the free-fall velocity $\dot{M}_{\rm ps} = M_{\rm disk}/t_{\rm ff}(R_{\rm acc})$, and (v) a **?**-type α-disk with some "effective viscosity," following **?** for gravito-turbulent viscosity and assuming the disk is truncated where $Q < 1$, giving $\dot{M}_{\rm ps} \sim (M_{\rm disk}/t_{\rm dep})\,(M_{\rm disk}/M_{\rm sink})^2$. We find the specific choice has little effect, except (i) gives artificially "bursty" accretion which produces spurious noise in the dust temperature; we adopt (v) as our default.

1.4.3 Protostellar Feedback from Sinks

(i) **Jets:** Protostars launch jets. We assume
???
(ii) **Dust Heating:** Radiation from protostars can heat dust; to model this we require the luminosity L of each protostar, which we take as the maximum of the accretion ($L_{\rm acc}$) and/or pre-main sequence luminosity ($L_{\rm pms}$). For protostars above the Deuterium burning limit, $M_{\rm ps} > 0.013\,M_{\odot}$, we take $L_{\rm acc} = 5\times10^{-7}\,\dot{M}_{\rm ps}\,c^2$, which corresponds to the standard linear size-mass relation with $R_{\rm ps} \sim 5\,(M_{\rm ps}/M_{\odot})$ (see e.g. **?**). Below the Deuterium burning limit, we include only the gravitational energy released by surface accretion, assuming a constant Jupiter-density size-mass relation, giving $L_{\rm acc} = 5\times10^{-8}\,(M_{\rm ps}/M_J)^{2/3}$. To be conservative, we take $L_{\rm pms}$ from a simple zero-age main sequence relation (likely to under-estimate the luminosity for
accretion to the surface of a Jupiter-density
???

(iii) **Radiation Pressure:**
???

1.5 "Star" Particles

1.5.1 Promotion

At the end of their pre-main sequence lifetime, we "promote" our proto-stellar particles to zero-age main sequence (ZAMS) stars. The pre-main sequence lifetime is calculated based on the Kelvin-Helmholtz time, with the simple toy model described in Appendix **??**. Directly using tabulated fits to stellar evolution models from MESA gives a similar result for our purposes. Roughly, the typical lifetimes scale as $\sim 50\,\mathrm{Myr}\,(M_{\rm ps}/M_{\odot})^{-2.5}$.

At this point, accretion and protostellar jets are terminated, we assume the remainder of the accretion disk is dispersed, and the stars begin to follow standard stellar evolution tracks.

1.5.2 Feedback

Stars act on the gas via several mechanisms:

(i) **Dust Heating:** The stars continue to heat dust; this is followed in the same manner as in § 1.4.3. We assume each star of mass M_* emits blackbody radiation with luminosity:

$$\frac{L_*}{L_{\odot}} = \begin{cases} 0 & (m < 0.012) \\ 0.185\,m^2 & (0.012 < m < 0.43) \\ m^4 & (0.43 < m < 2) \\ 1.5\,m^{3.5} & (2 < m < 53.9) \\ 32000\,m & (53.9 < m) \end{cases} \tag{3}$$

where $m \equiv M_*/M_{\odot}$; we assume a size-mass relation $R_* = R_{\odot}\,m^{\beta}$ with $\beta \approx 0.738$ to calculate the effective temperature $T_{\rm eff}(M_*)$.

(ii) **Radiation Pressure:** This also continues to act per § 1.4.3, but now using the ZAMS luminosity and spectrum.

(iii) **Photo-Electric Heating:** This is followed as in **?**. Given L_* and $T_{\rm eff}$, we estimate the production of photons from each star with energies $X - Y???\,\mathrm{eV}$, and follow their transport per § 1.2.6. This is used to define the local radiation field seen by each dust grain, which determines the heating rate following **?**.

(iv) **Photo-Ionization Heating:** This is followed as in **?**. Given L_* and $T_{\rm eff}$, we calculate the ionizing photon production rate Q from each star, and follow transport of these photons per § 1.2.6. This is combined with the metagalactic UV background as described in § 1.3 to self-consistently determine the photo-ionization heating rate and gas ionization state.

(v) **Winds:** ???? Stuff

$$\frac{\dot{M}_{\rm wind}}{M_{\odot}\,\mathrm{yr}^{-1}} = 2.34\times10^{-9}\,\frac{\alpha\,(q\,K)^{1/\alpha}}{q}\,\left(\frac{L_*}{L_{\odot}}\right)^{7/8}\,m^{0.185} \tag{4}$$

$$q \equiv \frac{(1-\alpha)\,\Gamma}{1-\Gamma} \tag{5}$$

$$\alpha \approx 0.5 + 0.4\,(1+16\,m^{-1})^{-1} \tag{6}$$

where $K \approx 1/30$ and the weak dependence of α on mass are calibrated to observations and $\Gamma = L_*/L_{\rm Edd}(M_*)$ is the Eddington factor.

(vi) **Cosmic Rays:** Following **?**, if we enable explicit tracking of cosmic rays (§ 1.2.5), then whenever fast winds ($v_{\rm wind} > 500\,\mathrm{km\,s^{-1}}$, though the exact choice is not important) and/or SNe are coupled to gas surrounding a star, we assume a fixed fraction $= 10\%$ of the kinetic energy goes into the cosmic ray component.

(vii) **Supernovae:** SNe are implemented following **?**. If a star with $M_* > 8\,M_{\odot}$ reaches the end of its main-sequence lifetime (simply estimated as $t_{\rm MS} \approx 9.6\,\mathrm{Gyr}\,(M_*/M_{\odot})\,(L_*/L_{\odot})^{-1}$), it explodes. The entire remaining mass goes into ejecta (i.e. we ignore relics), with kinetic energy 10^{51} erg, and metal yields calculated from **?**. The detailed coupling is numerically identical to that for winds.

APPENDIX A: NON-IDEAL FLUID TERMS

A1 Non-Ideal MHD

Astrophysical non-ideal MHD includes Ohmic dissipation, the Hall effect, and ambipolar diffusion. All appear as diffusion operators in the induction equation; if we operator-split the ideal MHD term (already solved in GIZMO), we have

$$\frac{d\mathbf{B}}{dt} = -\nabla\times\left[\eta_O\,\mathbf{J} + \eta_H\,\left(\mathbf{J}\times\hat{B}\right) - \eta_A\,\left(\mathbf{J}\times\hat{B}\right)\times\hat{B}\right] \tag{A1}$$

where $\mathbf{J} = \nabla\times\mathbf{B}$. The diffusivities $\eta_{O,\,H,\,A}$ govern Ohmic resistivity, the Hall effect, and ambipolar diffusion, respectively, and are given by the general expressions:

$$\eta_O \equiv \frac{c^2}{4\pi}\,\frac{1}{\sigma_O} \tag{A2}$$

$$\eta_H \equiv \frac{c^2}{4\pi}\,\frac{\sigma_H}{\sigma_H^2+\sigma_P^2} \tag{A3}$$

$$\eta_A \equiv \frac{c^2}{4\pi}\,\left[\frac{\sigma_P}{\sigma_H^2+\sigma_P^2} - \frac{1}{\sigma_O}\right] \tag{A4}$$

$$\sigma_O \equiv \frac{e\,c}{B}\,\sum_j n_j\,|Z_j|\,\beta_j \tag{A5}$$

$$\sigma_H \equiv \frac{e\,c}{B}\,\sum_j \frac{n_j\,Z_j}{1+\beta_j^2} \tag{A6}$$

$$\sigma_P \equiv \frac{e\,c}{B}\,\sum_j \frac{n_j\,|Z_j|\,\beta_j}{1+\beta_j^2} \tag{A7}$$

$$\beta_j \equiv \frac{|Z|_j\,e\,B}{m_j\,c\,\nu_j} \tag{A8}$$

where $\sigma_{O,\,H,\,P}$ are the Ohmic, Hall, and Pedersen conductivities, the index j sums over the different relevant species in the fluid (here ions, electrons, neutrals, and dust grains, e, i, n, g, respectively), $Z_n = 0$, $Z_e = -1$, $Z_i = +1$ and Z_g are the mean neutral/electron/ion/grain charges, $m_{n,\,e,\,i,\,g}$ the electron/ion/grain mass ($m_n = \mu\,m_p$ is calculated using the appropriate mean molecular weight μ for the element abundances, temperature, and molecular fraction determined in the cooling chemistry, and $m_g = (4\pi/3)\,a_g^3\,\bar{\rho}_g$ with a_g the grain radius and $\bar{\rho}_g$ the internal grain material density), $n_{n,\,e,\,i,\,g}$ the number density of each species (with mean number density $n = \rho/(\mu\,m_p)$ and $n_g = m_n\,f_{dg}\,n/m_g$ with f_{dg} the dust-to-gas ratio by mass), m_p is the proton mass, e the electron charge, c the speed of light, and $B = |\mathbf{B}|$ the magnitude of the magnetic field.

At the densities and temperatures of interest, the collision frequencies $\nu_{e,\,i,\,g}$ are given by:

$$\nu_e = 0.051\, n_e\, T_{100}^{-1.5}\,\mathrm{cm^3\,s^{-1}} + \frac{\rho_n\,\left[5.15\,T_{100}^{0.65} + 1.06\,T_{100}^{0.5}\right]}{10^9\,(m_n + m_e)}\,\mathrm{cm^3\,s^{-1}} \tag{A9}$$

$$\nu_i = 0.051\,\frac{\rho_e}{\rho_i}\,n_e\,T_{100}^{-1.5}\,\mathrm{cm^3\,s^{-1}} + \frac{\rho_n\,\left[1.91\,\left(\frac{m_p}{\mu_{\rm i-H_2}}\right)^{\frac{1}{2}} + 0.31\,\left(\frac{m_p}{\mu_{\rm i-He}}\right)^{\frac{1}{2}}\right]}{10^9\,(m_n + m_i)}\,\mathrm{cm^3\,s^{-1}} \tag{A10}$$

$$\nu_g = \frac{\pi\,a_g^2\,\delta_{gn}\,\rho_n}{(m_n + m_g)}\,\left(\frac{128\,k_B T}{9\pi\,m_n}\right)^{1/2} \tag{A11}$$

where the terms in ν_e and ν_i represent electron-ion, electron/ion-H_2, and electron/ion-He collisions, respectively, with assumed H, He abundances $\approx 0.76,\,0.24$ (changing this has negligible effects), and $T_{100} \equiv T/100\,K$; in ν_g, $\delta_{gn} \approx 1.3$ is the Epstein coefficient for spherical grains (**?**).

We follow **?** and assume the grains have a non-evolving size distribution, and that the system obeys global charge neutrality and local ionization equilibrium: this allows us to calculate

$$Z_g \equiv -\psi\,\frac{a_g\,k_B\,T}{e^2} \tag{A12}$$

$$\psi = \alpha\,\left(\exp\left(\psi\right) - \frac{(m_i/m_e)^{1/2}}{1+\psi}\right) \tag{A13}$$

$$\alpha \equiv \frac{\zeta\,e^2\,m_e^{1/2}\,m_g^2}{(8\pi)^{1/2}\,a_g^3\,f_{dg}^2\,(k_B T)^{3/2}\,m_n^2\,n} \tag{A14}$$

$$n_i = \frac{\zeta\,n}{k_{ig}\,n_g} \tag{A15}$$

$$n_e = \frac{\zeta\,n}{k_{eg}\,n_g} \tag{A16}$$

$$k_{ig} \equiv \pi\,a_g^2\,\left(\frac{8\,k_B\,T}{\pi\,m_i}\right)^{1/2}\,(1+\psi) \tag{A17}$$

$$k_{eg} \equiv \pi\,a_g^2\,\left(\frac{8\,k_B\,T}{\pi\,m_e}\right)^{1/2}\,\exp\left(-\psi\right) \tag{A18}$$

where k_B is the Boltzmann constant, T is the gas kinetic temperature, $k_{ig,\,eg}$ are the coefficients for ion-grain and electron-grain collisions respectively.

For the grains, we assume an effective size $a_g = 0.1\,\mu m$, material density $\bar{\rho}_g = 3\,\mathrm{g\,cm^{-3}}$ (typical of both silicate and carbonaceous grains), and constant dust-to-metals ratio $f_{dg} = 0.01\,(Z/Z_{\odot})$. For reasonable values, these choices only weakly influence the coefficients $\eta_{O,H,A}$, compared to the values of n, T, and ζ. We assume that at low temperatures and high densities, the ions are dominated by Mg, so $m_i = 24.3\,m_p$ (at high temperatures, lighter ions become important, but this only changes the diffusivities where they are dynamically irrelevant; explicitly assuming $m_i = m_p$ above 200 K, for example, we obtain identical conclusions). This completely determines the non-ideal MHD coefficients, given the MHD state of the gas and ζ.

A2 Spitzer-Braginskii Viscosity and Conduction

As detailed in **?**, the implementation of Spitzer-Braginskii viscosity and conduction in GIZMO adds the following additional energy and momentum fluxes to the standard MHD equations:

$$\mathbf{F}_e^{SB} = \kappa\,\hat{B}\,\left(\hat{B}\cdot\nabla u\right) + \Pi\cdot\mathbf{v} \tag{A19}$$

$$\mathbf{F}_p^{SB} = \Pi \equiv 3\,\nu\,\mathbf{K}\,\left[\mathbf{K} : (\nabla\otimes\mathbf{v})\right] \tag{A20}$$

$$\mathbf{K} \equiv \hat{B}\otimes\hat{B} - \frac{1}{3}\mathbf{I} \tag{A21}$$

$$\kappa = \frac{0.96\,(k_B T)^{5/2}}{m_e^{1/2}e^4\ln\Lambda}\,\left(1 + 4.2\,\ell_e/\ell_T\right)^{-1} \tag{A22}$$

$$\nu = \frac{0.406\,m_i^{1/2}\,(k_B T)^{5/2}}{(Z_i\,e)^4\ln\Lambda}\,\left(1 + 4.2\,\ell_e/\ell_T\right)^{-1} \tag{A23}$$

where $\otimes$ denotes the outer product, $\hat{B}$ is the direction of the magnetic field vector, $\mathbf{I}$ is the identity matrix, $\mathbf{v}$ the velocity, u the specific internal energy, : denotes the double-dot-product ($\mathbf{A} : \mathbf{B} \equiv \mathrm{Trace}(\mathbf{A}\cdot\mathbf{B})$), $\ln\Lambda \approx 37.8$ is the Coulomb logarithm (**?**), ℓ_e is the electron mean-free path, and $\ell_T = T/|\nabla T|$ is the temperature gradient scale length. In these equations, κ is the conductivity, and ν the viscosity (with Π the viscous tensor). Details of the coefficients are in (Su et al., in prep.).

A3 Turbulent Diffusivity

In some "sub-grid" models for turbulence, the effects of unresolved eddies are treated as diffusion processes. Following **?**, we can approximate the "eddy diffusivity" as

$$D \equiv (C\,\Delta x)^2\,\|\mathbf{S}\| \tag{A24}$$

where $C \approx 0.15$ is a constant calibrated to numerical simulations in **?**, Δx is the grid scale (for our MFM method, this is equal to the rms inter-element spacing), and $\mathbf{S} \equiv [(\nabla\otimes\mathbf{v}) + (\nabla\otimes\mathbf{v})^T] - \mathrm{Trace}(\nabla\otimes\mathbf{v})/3$ is the symmetric shear tensor.

With this enabled, we can treat sub-grid diffusion of internal energy and momentum using our standard conduction and viscosity equations but adding this "turbulent diffusivity" to the physical conductivity and viscosity. Passive scalars, in particular metals, are also diffused, even where there is no mass flux, with $\partial Z/\partial t = -\nabla\cdot(D\,\nabla Z)$. Since the abundance of ions and electrons is determined every timestep from the element MHD properties assuming local equilibrium, we do not need to explicitly treat their diffusion.

APPENDIX B: DUST DYNAMICS

In **???**, we describe in detail the numerical implementation and algorithmic as well as physical tests of explicit dust dynamics. Briefly, the dust is treated as a collection of "super-particles," each one of which represents a collection of grains of similar size whose trajectory is explicitly integrated in the code according to the equations of motion for grains of that size. We explicitly include drag, Lorentz, radiation pressure, and gravitational forces, i.e.

$$\frac{d\mathbf{v}_g}{dt} = \mathbf{a}_{\rm drag} + \mathbf{a}_{\rm Lor} + \mathbf{a}_{\rm rad} + \mathbf{a}_{\rm grav} \tag{B1}$$

$$\mathbf{a}_{\rm drag} = -\frac{(\mathbf{v}_g - \mathbf{u}_{\rm gas})}{t_s} \tag{B2}$$

$$t_s \equiv \frac{\pi^{1/2}}{8^{1/2}}\,\frac{\bar{\rho}_g\,a_g}{c_s\,\rho_{\rm gas}}\,\left[\left(1+\frac{9\pi\,s^2}{64}\right)^{\frac{1}{2}} + \frac{\alpha_c\,(n_i/n)}{1+\frac{4\,s^3}{3\pi^{1/2}}}\right]^{-1}$$

$$\mathbf{a}_{\rm Lor} = \frac{Z_g\,e}{m_g\,c}\,(\mathbf{v}_g - \mathbf{u}_{\rm gas})\times\mathbf{B} \tag{B3}$$

$$\mathbf{a}_{\rm rad} = \frac{\pi\,a_g^2\,\tilde{Q}}{m_g\,c}\,\mathbf{F}_{\rm rad} \tag{B4}$$

where $d\mathbf{v}_g/dt$ is a Lagrangian derivative, $\mathbf{v}_g$ the grain velocity, $\mathbf{u}_g$ the gas velocity, $s \equiv |\mathbf{v}_g - \mathbf{u}_{\rm gas}|/c_s$, Z_g is the grain charge (determine as described in § A1), $\mathbf{F}_{\rm rad}$ the net incident radiation flux (computed as described in the text), and $\tilde{Q}$ is the dimensionless, flux-integrated absorption efficiency which we take to be unity for simplicity. The second term in t_s accounts for Coulomb interactions (with $\alpha_c \approx 11.2$ from **?**); we include it for completeness but it is only important at temperatures $T \gg 10^4$ K. The gravitational force $\mathbf{a}_{\rm grav}$ (and self-gravity from dust) is treated identically to our other particle types.

We do not explicitly follow grain formation, destruction, or size evolution, since these are both highly uncertain and, in most models, produce evolution on longer timescales than we are interested in here. However we will study how the grain dynamics can modify these evolution equations on longer timescales. Dust-dust collisions are always subdominant to dust-gas drag in the dynamics equations for the dust (although they can be important for dust size evolution). And we ignore back reaction from the dust on the gas, which is only important when the local dust-to-gas ratio exceeds unity.

We assume a constant grain material density $\bar{\rho}_g = 3\,{\rm g\,cm^{-3}}$ and populate a grain size distribution from $\sim 0.1 - 10\,\mu m$. Smaller grains are very tightly coupled and we assume they move with the gas.

APPENDIX C: COSMIC RAYS

A detailed series of papers on our numerical treatment of CRs and their consequences for galaxy and star formation will be the subject of future work (in preparation). Here we briefly outline the key physical equations being solved, as, for our implementation here, these have all appeared in previous work.

CRs are approximated as a single-species, ultra-relativistic ($\gamma_{cr} = 4/3$) fluid. In the code we evolve the conserved variable $E_{{\rm cr},\,i}$, the total CR energy associated with particle i. We follow **??** in our implementation, with some significant improvements. Following **?**, the evolution equation for the CR energy density e_{cr} is

$$\frac{\partial e_{cr}}{\partial t} = (\mathbf{v}+\mathbf{v}_{\rm st})\cdot\nabla P_{\rm cr} - \nabla\cdot\left[(\mathbf{v}+\mathbf{v}_{\rm st})\,(e_{cr}+P_{cr})\right] + \nabla\cdot\left[\mathbf{v}_{di}\,e_{cr}\right] - \Gamma_{cr} + \dot{e}_* \tag{C1}$$

$$P_{cr} = (\gamma_{cr}-1)\,e_{cr} \tag{C2}$$

$$\mathbf{v}_{st} = -\left(c_s^2+v_A^2\right)^{1/2}\,\hat{B}\,\left(\frac{\hat{B}\cdot\nabla P_{cr}}{|\nabla P_{cr}|}\right) \tag{C3}$$

$$\mathbf{v}_{di} = \kappa_{di}\,\hat{B}\,\left(\frac{\hat{B}\cdot\nabla e_{cr}}{e_{cr}}\right) \tag{C4}$$

$$\Gamma_{cr} = 7.51\times10^{-16}\,{\rm s^{-1}}e_{cr}\,(1+0.22\,\tilde{n}_e)\,\left(\frac{n_H}{\rm cm^{-3}}\right) \tag{C5}$$

$$\kappa_{di} \sim \frac{v_{cr}\,r_g}{3}\,\frac{B^2_{\rm coherent}}{B^2_{\rm random}(r_g)} \sim 3\,R_{GV}^{1/3}\times10^{28}\,\frac{\rm cm^2}{\rm s}\,\left(\frac{\mu G}{|\mathbf{B}|}\right)^{\frac{1}{3}}\,\left(\frac{L_{\rm drive}}{\rm kpc}\right)^{\frac{2}{3}} \tag{C6}$$

where P_{cr} is the CR pressure, $\mathbf{v}$ is the gas speed, $\mathbf{v}_{st}$ is the CR streaming speed, c_s the sound speed, v_A the Alfven velocity (so $|\mathbf{v}_{st}|$ scales as c_s when $v_A \ll c_s$ and v_A when $c_s \ll v_A$), $\hat{B} = \mathbf{B}/|\mathbf{B}|$ where $\mathbf{B}$ is the magnetic field vector, $\mathbf{v}_{di}$ is a diffusion speed with κ_{di} the CR diffusion coefficient.

The first terms here includes advection and adiabatic effects; advection with gas follows trivially from our Lagrangian code; adiabatic CR compression/expansion, and work done on the gas, are accounted for self-consistently in the Riemann problem. The streaming terms account for the CR streaming instability; in this form the streaming terms resemble a diffusion equation and we solve them as such. Note that the $\mathbf{v}_{st}\cdot\nabla P_{cr}$ term represents heating due to self-excited waves that are rapidly damped in the plasma, which produce an energy loss from the CRs which we assume is rapidly thermalized so is added as a gas-heating term.

The diffusion coefficient κ_{di} is more uncertain; we have also considered adopting a simple constant, Milky Way-like value of $\kappa_{di} \sim 3\times10^{28}\,{\rm cm^2\,s^{-1}}$, and find this makes no difference to our major conclusions. This is not surprising given that CRs do not qualitatively alter our results or appear to dominate the feedback. But for our default we use the standard turbulent diffusivity; here $B_{\rm coherent}$ and $B_{\rm random}(r_g)$ represent the coherent mean $\mathbf{B}$-field and random component on the scale r_g of the gyro radius of CRs; assuming $v_{cr} \sim c$, a constant $R_{GV} \sim 1$ magnetic rigidity in giga-volts (which determines r_g), and a Kolmogorov spectrum for $B^2_{\rm random}$ with driving scale of order the pressure gradient scale length ($L_{\rm drive} \sim P/|\nabla P|$), we obtain the second expression for κ_{di}, which is used in the simulations here.

Γ_{cr} is the loss rate of CRs to gas and radiation, here from **?** including combined hadronic plus Coulomb losses; n_H is the hydrogen number density and $\tilde{n}_e$ is the number of free electrons per hydrogen nucleus. Following their estimate, 1/6 of the hadronic losses and all of the Coulomb losses are assumed to be thermalized and appear as a gas heating term. This makes it clear how e_{cr} is directly related to the cosmic ray ionization rate ζ defined in the text and needed for ionization calculations: for our definitions $\zeta \approx 10^{-17}\,{\rm s^{-1}}\,[e_{cr}/1.3\,{\rm eV\,cm^{-3}}]$. Finally, $\dot{e}_*$ represents injection via stellar feedback (described above).

Note that the equations here include the proper coupling of the CRs to magnetic fields. Unlike many previous studies, we include this and solve the fully anisotropic diffusion equations. A detailed comparison of numerical methods for anisotropic diffusion in mesh-free codes is in preparation, but the results in GIZMO agree very well, on all tests, with those from ATHENA.

APPENDIX D: LUMINOSITIES

D1 Protostars

The moment a sink particle is formed, a "protostar" of zero mass and zero size is created at its center. The protostar then evolves continuously to higher masses and non-zero sizes as it accretes mass from the "reservoir" (described in the text) of mass which has been swallowed by the sink.

Each such protostar begins on the Hayashi track. The mass evolves only via accretion. The size evolves according to a combination of accretion and contraction. If the star is accreting sufficiently rapidly, it will increase in radius owing to the new material – however as soon as accretion slows down, contraction takes over. For simplicity, on the Hayashi track (neglecting accretion) we assume contraction at constant effective temperature. Eventually the star contracts to a radius $R_{\rm crit}$, where it reaches the Henyey track, from which point it contracts at approximately constant luminosity. Eventually it contracts to the zero-age main sequence (ZAMS) size, at which point we "promote" the proto-star (sink) to a zero-age "star."

At any point during the protostellar phase, the luminosity is the sum of accretion & internal luminosities:

$$L_{\rm ps} \equiv L_{\rm acc} + L_{\rm internal} \tag{D1}$$

where the accretion luminosity is given by

$$L_{\rm acc} \equiv \epsilon_r\, \dot{M}_{\rm ps}\, c^2 \tag{D2}$$

$$\epsilon_r = \begin{cases} 5\times10^{-7} & (m \ge 0.012) \\ 5\times10^{-8}\,(M_{\rm ps}/M_J)^{2/3} & (m < 0.012) \end{cases}$$

$$m \equiv M_{\rm ps}/M_\odot$$

and the internal luminosity depends on whether the protostar is on the Hayashi track

$$L_{\rm internal} = \begin{cases} L_{\rm Hayashi} & (R_{\rm ps} > R_{\rm crit}) \\ L_{\rm Henyey} & (R_{\rm ps} \le R_{\rm crit}) \end{cases} \tag{D3}$$

$$L_{\rm Henyey} \approx L_{\rm ZAMS}(M_{\rm ps}) \tag{D4}$$

$$\frac{L_{\rm ZAMS}}{L_\odot} = \begin{cases} 0 & (m < 0.012) \\ 0.185\,m^2 & (0.012 < m < 0.43) \\ m^4 & (0.43 < m < 2) \\ 1.5\,m^{3.5} & (2 < m < 53.9) \\ 32000\,m & (53.9 < m) \end{cases}$$

$$L_{\rm Hayashi} \approx L_{\rm KH} = 0.226\,L_\odot \left(\frac{R_{\rm ps}}{R_\odot}\right)^2 m^{0.55} \tag{D5}$$

$$R_{\rm crit} \equiv R_{\rm ps}[L_{\rm Hayashi} = L_{\rm Henyey}\,|\,m] \tag{D6}$$

The luminosity at each time is taken to be a blackbody with effective temperature $T_{\rm eff}^4 = L_{\rm ps}/(4\pi\,\sigma_B\,R_{\rm ps}^2)$.

The protostellar radii are evolved explicitly for each protostar, according to contraction and new accretion (where the new material comes in with large radius):

$$\dot{R}_{\rm ps} = -\frac{R_{\rm ps}}{t_{\rm c}(R_{\rm ps},\,...)} + (R_0 - R)\,\frac{\dot{M}_{\rm ps}}{M_{\rm ps}} \tag{D7}$$

where R_0 is an arbitrary (large) initial radius (which has almost no effect on our results, since so long as it is large, initial contraction of new material is very fast), taken to be $R_0 \equiv 100\,R_\odot\,m$, and $t_{\rm c}$ is the contraction time. On the Hayashi track ($R_{\rm ps} > R_{\rm crit}$), we assume contraction at constant effective temperature, $T_{\rm eff} = 4000\,{\rm K}\,m^{0.55}$, and on the Henyey track ($R_{\rm ps} \le R_{\rm crit}$), contraction at constant luminosity $L_{\rm Henyey}$, giving:

$$t_{\rm c} = \begin{cases} 80.21\,{\rm Myr}\,m^{1.45}\,r_{\rm ps}^{-3} & (R_{\rm ps} > R_{\rm crit}) \\ 18.15\,{\rm Myr}\,m^2\,r_{\rm ps}^{-1}\left(\dfrac{L_\odot}{L_{\rm Henyey}}\right) & (R_{\rm ps} \le R_{\rm crit}) \end{cases} \tag{D8}$$

$$r_{\rm ps} \equiv R_{\rm ps}/R_\odot$$

We consider the proto-star to reach the main-sequence "ignition" radius (where contraction should halt) when we reach the radius $r_{\rm ps} \le m^{0.8}$ for $m < 1$ or $r_{\rm ps} \le m^{0.57}$ for $m \ge 1$. At that point we promote the proto-star to a "star."

D2 Main-Sequence Stars

Once on the main sequence, stars are not allowed to accrete, and are assigned a luminosity $L_* = L_{\rm ZAMS}$ (Eq. D4).