# 1 Expressions

¬p∨p vs. $\neg p \lor p$
¬(p∧¬p) vs. $\neg(p \land \neg p)$
$(p \to (q \land r)) \to (q \land r \land p)$
$(q.r) \supset (q \supset r)$

## 2 Evaluation

$$
\begin{array}{ccc}
p & \to & \bot \\
q & \to & \top \\
\hline
p & \to & 0 \\
q & \to & 1 \\
\hline
p & \to & F \\
q & \to & T
\end{array}
$$

$$
\begin{array}{ccc}
\hline
p & \to & \bot \\
q & \to & \top \\
\hline
p & \to & \bot \\
q & \to & \top \\
r & \to & \top \\
\hline
\end{array}
$$

$$
\begin{array}{ccc}
p & \to & \bot \\
q & \to & \top \\
r & \to & \bot \\
s & \to & \top \\
\hline
\end{array}
$$

| a | b | c | d |
|---|---|---|---|
| $p \to \bot$ | $p \to F$ | $p \to \top$ | $p \to 1$ |
| $q \to \bot$ | $q \to T$ | $q \to \bot$ | $q \to 1$ |
| $p \vee q \to \bot$ | $p \vee q \to T$ | $p \vee q \to \top$ | $p \vee q \to 1$ |
| $p \wedge q \to \bot$ | $p \wedge q \to F$ | $p \wedge q \to \bot$ | $p \wedge q \to 1$ |
| $p \to q \to \top$ | $p \to q \to T$ | $p \to q \to \bot$ | $p \to q \to 1$ |
| $p \to (\neg q) \to \top$ | $p \to (\neg q) \to T$ | $p \to (\neg q) \to \top$ | $p \to (\neg q) \to 0$ |
| $(\neg p) \to q \to \bot$ | $(\neg p) \to q \to T$ | $(\neg p) \to q \to \top$ | $(\neg p) \to q \to 1$ |
| $p \wedge \neg q \to \bot$ | $p \wedge \neg q \to F$ | $p \wedge \neg q \to \top$ | $p \wedge \neg q \to 0$ |
| $\neg p \wedge q \to \bot$ | $\neg p \wedge q \to T$ | $\neg p \wedge q \to \bot$ | $\neg p \wedge q \to 0$ |

## 3 Truth tables

| ∧ | p | q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 1 | 1 | 0 |
| 0 | 1 | 1 |

| p | ∧ | ¬ | q |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |

| p | ∧ | ¬ | q |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |

| ((p | → | q) | → | (r | → | (p | → | q) | → | (r | → | q))) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| p | ∧ | (¬ | q | → | r) |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 |

```
1   0   1   0   0   0
1   1   1   0   1   1
1   1   0   1   1   0
1   1   0   1   1   1
```

# 4  Syntax trees

```
        ⟨∧⟩
       ╱    ╲
    ⟨p⟩      ⟨¬⟩
              │
             ⟨q⟩
```

| → | *p* | *q* |
|---|-----|-----|
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

```
        ┌───┐
        │ → │
        └───┘
        ╱     ╲
     ╱           ╲
 ┌───┐          ┌───┐
 │ p │          │ ∨ │
 └───┘          └───┘
              ╱       ╲
           ╱             ╲
       ┌───┐             ┌───┐
       │ ¬ │             │ q │
       └───┘             └───┘
         │
       ┌───┐
       │ p │
       └───┘
```

```
        ∧
       / \
      p   ¬
           |
           q
```

```
        ∧
       / \
      p   ¬
           |
           q
```

∧

p  ¬

q
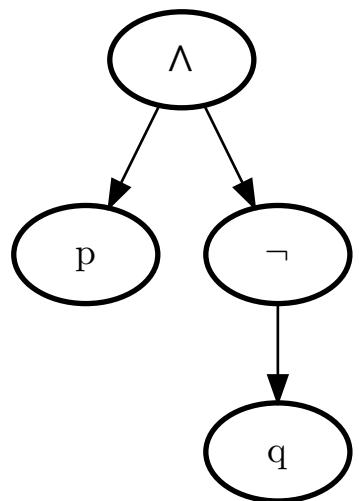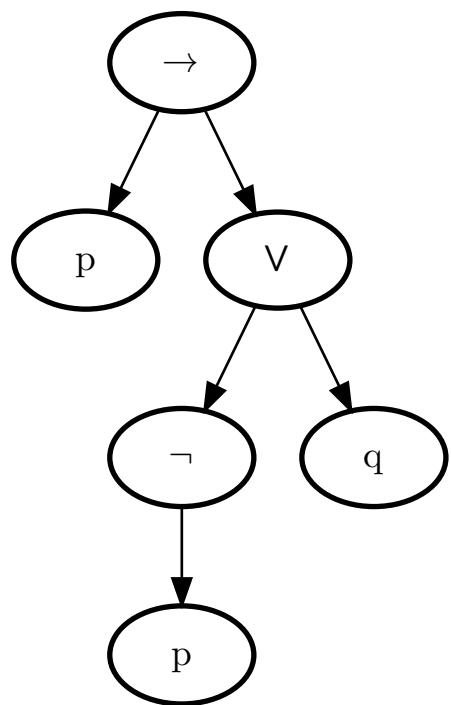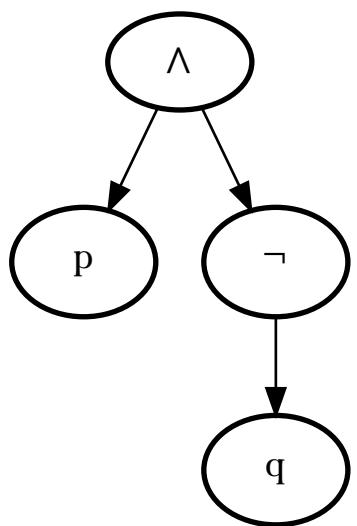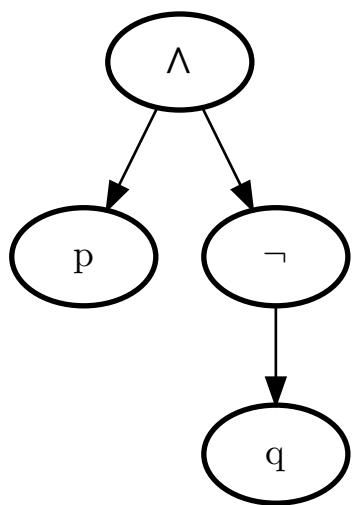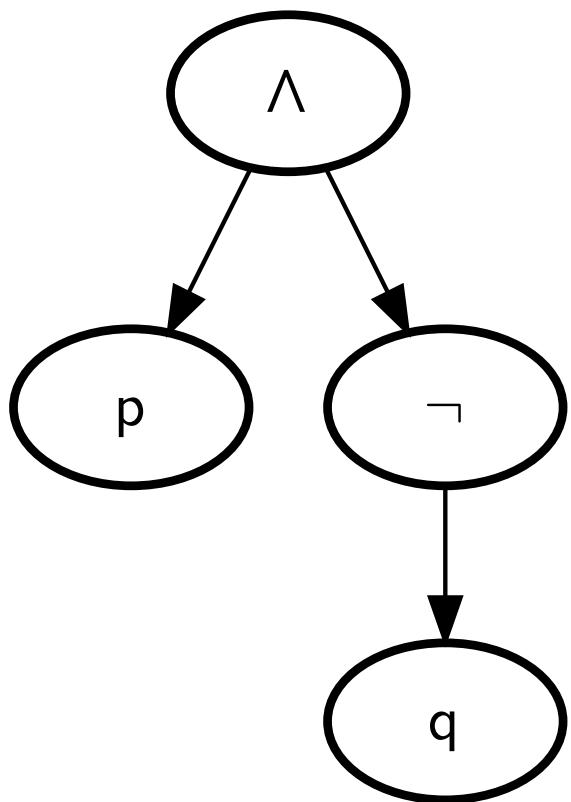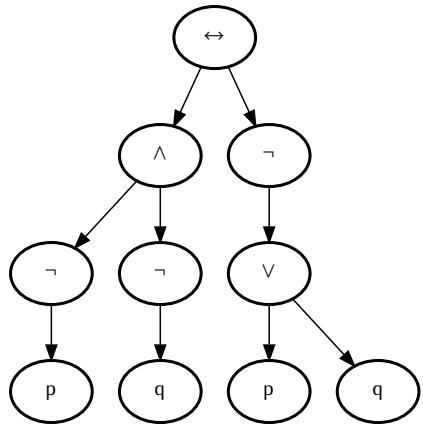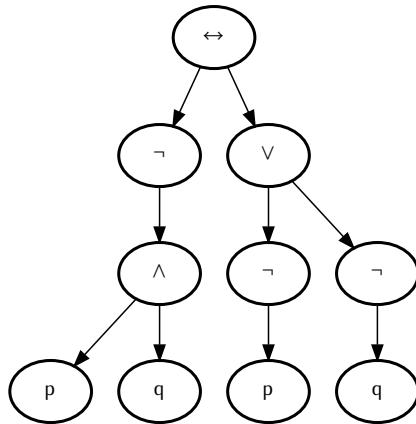
DeMorgan's law 1



DeMorgan's law 2

| $\leftrightarrow$ | $p$ | $q$ |
|---|---|---|
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

Truth
table 1

| $\leftrightarrow$ | $p$ | $q$ |
|---|---|---|
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

Truth
table 2

# 5 Venn diagrams
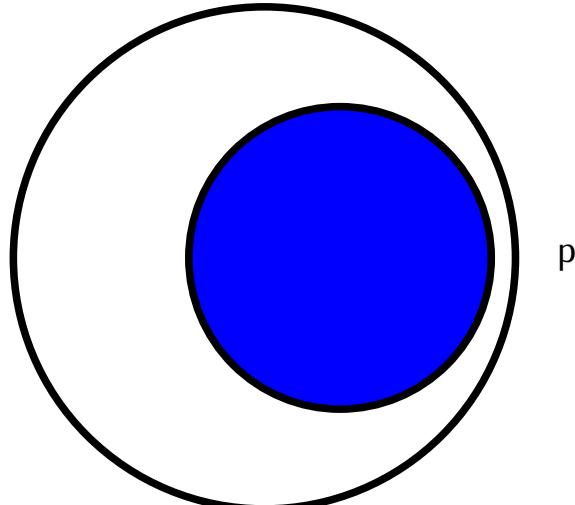


**Figure 1**  $\neg p$
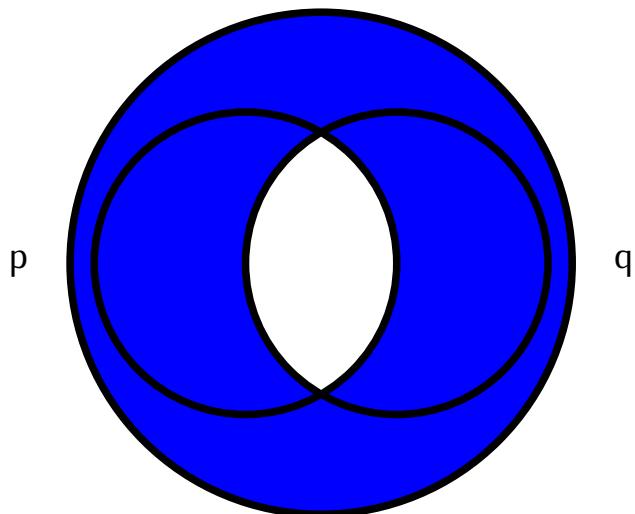


**Figure 2**  $p \wedge q$
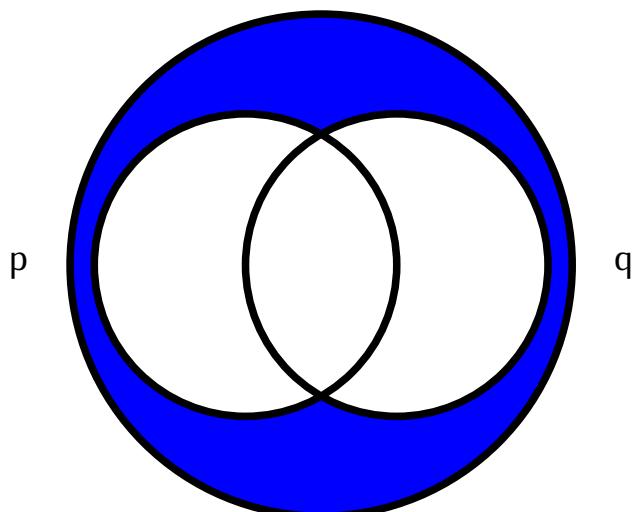
**Figure 3** $p \lor q$
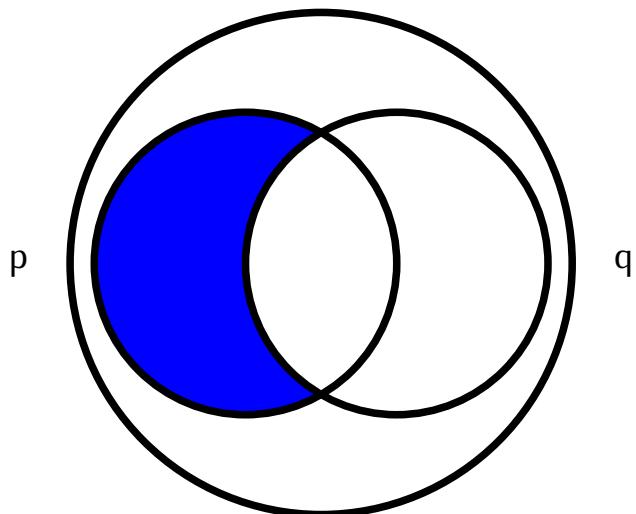


**Figure 4** $p \supset q$
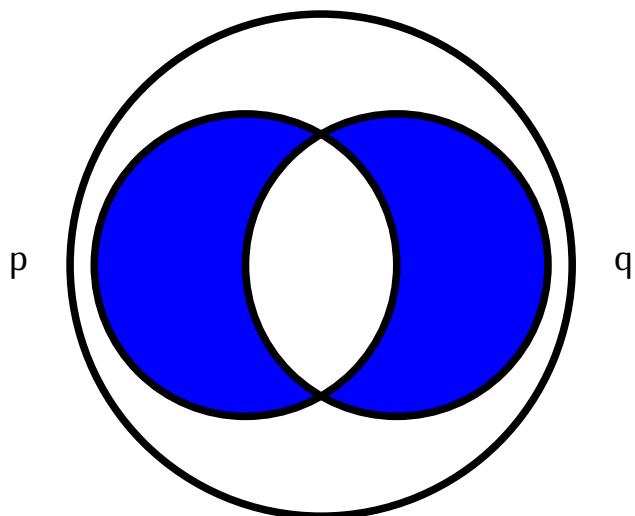
**Figure 5**   $p \leftrightarrow q$



**Figure 6**   $p \rightarrow (q \wedge r)$

**Figure 7**   $(p \rightarrow q) \wedge r$