It's an Economic Small-World

Big Day In

Macquarie University

12–13 February 2009

Minh Van Nguyen

nguyenminh2@gmail.com

Vacation Scholar 2008/2009

Mathematical and Information Sciences

Outline

- Do you Facebook: social networks
- Small-world networks
- It's about efficiency: economic small-worlds
- My contributions
- Conclusion & further research

CSIRO

Do you Facebook: social networks

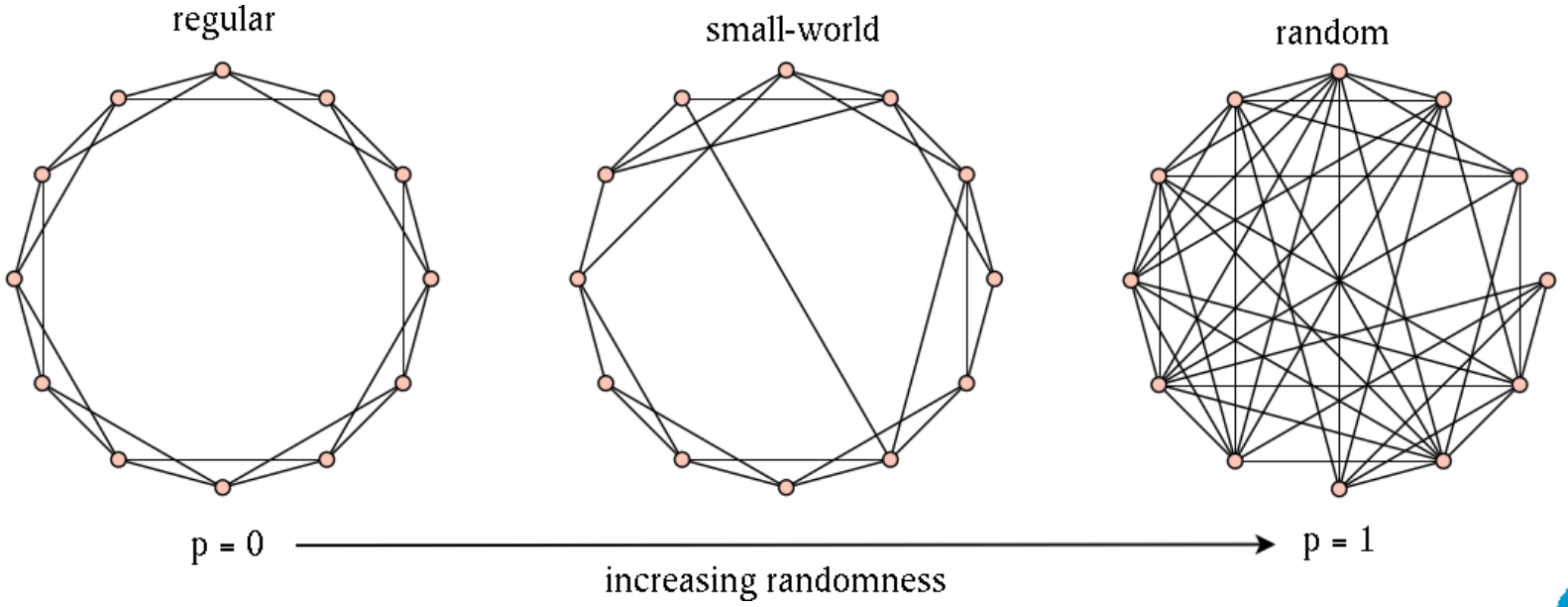
- Online social networking: Facebook [4], MySpace [9], Bebo [1]

- Network of movie actors
  - What's your Kevin Bacon number?
- Network of scientific collaboration
  - What's your Erdös number [2]?
- Economics network
  - What's your coffee bean number?

CSIRO

Small-world networks

- Watts-Strogatz [12] small-world model
  1. Start with a ring lattice.
  2. Random edge rewiring with probability p.
  3. Result: a small-world network somewhere between regular and Erdös-Rényi random [3]. (Figure produced using Sage [11] and NetworkX [5].)



CSIRO

Small-world networks

- Watts-Strogatz [12] small-world effect
  - high cliquishness C; low characteristic path length L

$$L(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in V(G)} d_{ij} \tag{1}$$

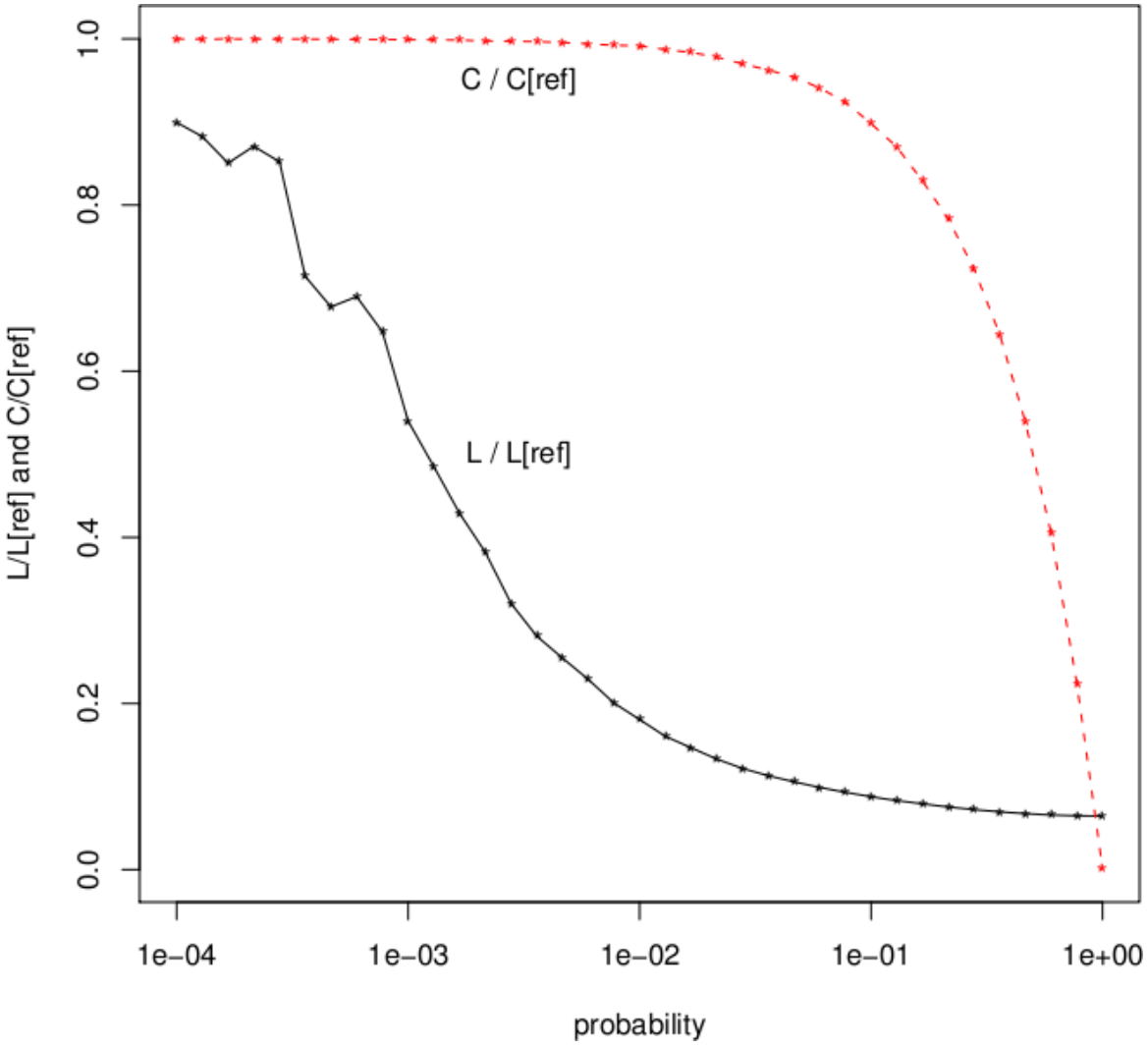
$$C(G) = \frac{1}{N} \sum_{i \in V(G)} C_i \qquad \text{where} \tag{2}$$

$$C_i = \frac{K_i}{N_i(N_i-1)/2} \tag{3}$$

CSIRO

Small-world networks

- Normalized plot on log scale (Watts-Strogatz). Figure produced using R [10].



CSIRO

It's about efficiency: economic small-worlds

- Latora-Marchiori [8] generalization: economic small-worlds
  - global efficiency E_{glob}; local efficiency E_{loc}
  - network cost C_G

$$E_{glob} = E(G) \Big/ E(\kappa_N) \tag{4}$$

$$E_{loc} = \frac{1}{N} \sum_{i \in V(G)} \frac{E(G_i)}{E(\kappa_{|V_i|})} \tag{5}$$

$$C_G = \sum_{i \neq j \in V(G)} a_{ij} \gamma(\ell_{ij}) \Big/ \sum_{i \neq j \in V(G)} \gamma(\ell_{ij}) \qquad \text{where} \tag{6}$$

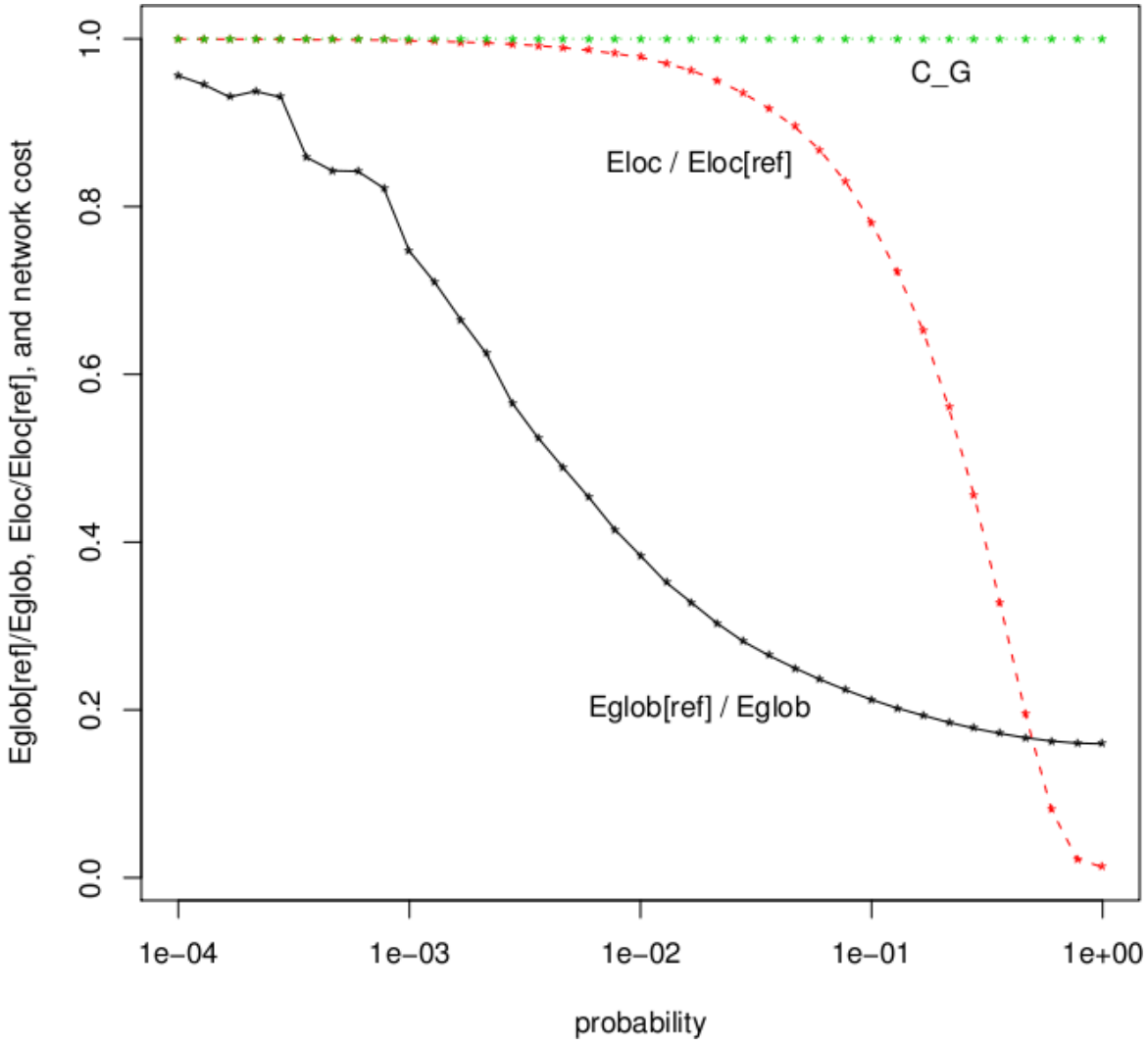
$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in V(G)} \frac{1}{d_{ij}} \qquad \text{and} \tag{7}$$

$$\ell_{ij} = \frac{\sin(|i-j|\pi/N)}{\sin(\pi/N)} \tag{8}$$

CSIRO

It's about efficiency: economic small-worlds

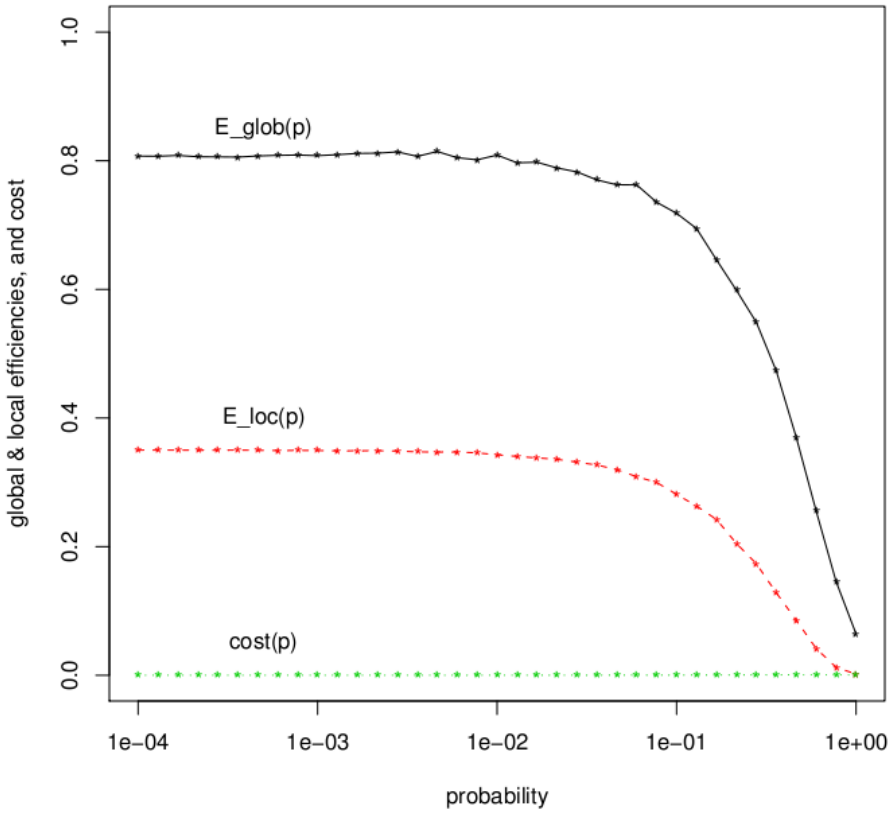
- Normalized plot on log scale (Latora-Marchiori, *unweighted*). Figure produced using R [10].

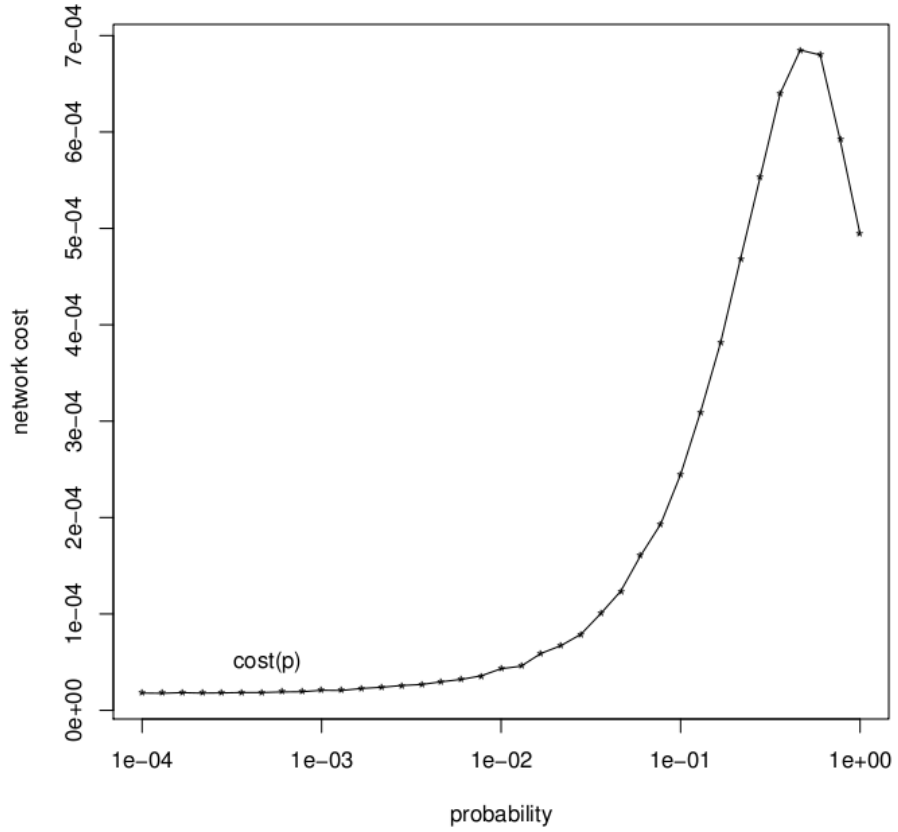


CSIRO

It's about efficiency: economic small-worlds

- Plotting metrics as functions of rewiring probabilities (Latora-Marchiori, *weighted*). Figure produced using R [10].





CSIRO

My contributions

- Verify results reported by Latora & Marchiori [8]
  - Using R environment for statistical computing [10]
  - Computational problems:
    - Was it R (i.e. the "sna" package)?
    - Better hardware: CSIRO's Burnet compute cluster
- Alternatives to using R
  - NetworkX [5]
  - Sage [11]
- Contributions to open source projects
  - NetworkX: developed connected variant of Watts-Strogatz model (upcoming NetworkX version 1.0)
  - Sage: contribute to updating Sage $\longleftrightarrow$ NetworkX interface package (upcoming Sage version 3.3)

CSIRO

Conclusion & further research

- Network approach to economics
  - social networks $\longrightarrow$ small-worlds $\longrightarrow$ economic small-worlds
- Multi-agent simulation
  - network of buyers & sellers [7]
  - edge weight: Cobb-Douglas or constant elasticity of substitution functions [6]
  - generalize to multiple input/output

CSIRO

Thank you

- Dr Rodolfo Garcia-Flores
- CSIRO
- CMIS staff

Questions?

CSIRO

References

[1] Bebo, Inc., 10 February 2009. http://www.bebo.com.

[2] R. D. Castro and J. W. Grossman. Famous trails to Paul Erdös. *Mathematical Intelligencer*, 21:51–63, 1999.

[3] P. Erdös and A. Rényi. On random graphs. *Publicationes Mathematicae*, 6:290–297, 1959.

[4] Facebook, 10 February 2009. http://www.facebook.com.

[5] A. A. Hagberg, D. A. Schult, and P. J. Swart. Exploring network structure, dynamics, and functions using NetworkX. In G. Varoquaux, T. Vaught, and J. Millman, editors, *Proceedings of the 7th Python in Science Conference (SciPy2008)*, pages 11–15, Pasadena, CA, USA, 2008. http://networkx.lanl.gov.

[6] M. D. Intriligator. *Mathematical Optimization and Economic Theory*. Prentice-Hall, Englewood Cliffs, NJ, USA, 1971.

[7] T. Kaihara. Multi-agent based supply chain modelling with dynamic environment. *International Journal of Production Economics*, 85:263–269, 2003.

[8] V. Latora and M. Marchiori. Economic small-world behavior in weighted networks. *The European Physical Journal B*, 32(2):249–263, 2003.

[9] MySpace, 10 February 2009. http://www.myspace.com.

[10] R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2009. ISBN 3-900051-07-0, http://www.r-project.org.

[11] W. A. Stein et al. *Sage Mathematics Software (Version 3.2.3)*. The Sage Development Team, 2009. http://www.sagemath.org.

[12] D. J. Watts and S. H. Strogatz. Collective dynamics of "small-world" networks. *Nature*, 393:440–442, 1998.